Class Drill 21: Using the Graphing Strategy to Graph a Polynomial

The goal is to graph the function $f(x) = x^3 - 3x^2 - 9x - 5 = (x + 1)^2(x - 5)$.

Step 1. Analyze $f(x)$.
- Find the $y$-intercept and the $x$-intercepts.
- Determine the end-behavior.
- Make a sign chart for $f$ and use it to determine where $f$ is positive, negative, or zero.

Step 2. Analyze $f'(x)$.
- Find $f'(x)$, factor it, and then find the partition numbers for $f'(x)$.
- Construct a sign chart for $f'(x)$ and use it to determine the intervals on which $f$ is increasing and decreasing, and to find the $x$-coordinates of all local maxima and minima.
- Find the $y$-coordinates of all local maxima and minima.
Step 3. Analyze $f''(x)$.
- Find $f''(x)$, factor it, and then find the partition numbers for $f''(x)$.
- Construct a sign chart for $f''(x)$ and use it to determine the intervals on which $f$ is concave up and concave down, and to find the $x$-coordinates of all inflection points.
- Find the $y$-coordinates of all inflection points.

Step 4: Sketch the graph of $f$.
- Plot the axis intercepts, local maxima and minima, and inflection points, and label them with their $(x,y)$ coordinates.
- Using the other information from steps 1, 2, and 3, draw the graph.