New Rule of Inference: The Principle of Induction

\[ P(a) \text{ is true} \]
For all integers \( k \geq a \), if \( P(k) \) is true, then \( P(k + 1) \) is true.
\[ \therefore \text{For all integers } n \geq a, \ P(n) \text{ is true.} \]

Usage:
- The letter “a” represents some fixed integer.
- The letters “k” and “n” represent variables whose domain \( D \) is the set of all integers greater than or equal to “a”.
- The symbol \( P(n) \) represents a predicate whose domain is the set \( D \).

This new rule of inference will be used to prove statements of the form

Statement \( S \): “For all integers \( n \geq a \), \( P(n) \) is true.”

Strategy for using the principle of induction

Preliminary work:
- Identify the number playing the role of “a”. (Introduce it.)
- Identify the predicate \( P(n) \). (Introduce it in a sentence.)
- Figure out what the expressions for \( P(a) \), \( P(k) \) and \( P(k + 1) \) look like. (Write them down.)

Build a proof of Statement \( S \) using the following structure:

Proof of Statement \( S \):
- Basis Step: Prove that \( P(a) \) is true.
  (1) The proof will begin somehow.
    * 
    * A bunch of steps may be involved. Usually a computation.
    * 
    (xx) \( P(a) \) is true.
- Inductive Step: Prove that for all integers \( k \geq a \), if \( P(k) \) is true, then \( P(k + 1) \) is true.
  (1) Suppose that \( k \) is an integer such that \( k \geq a \) and that \( P(k) \) is true.
    * 
    * a bunch of steps may be involved
    * 
    (xx) \( P(k + 1) \) is true. (some justification goes here.)
- Conclusion: Therefore, for all integers \( n \geq a \), \( P(n) \) is true. (by the principle of induction)

End of Proof of Statement \( S \)