Problem: 1  2  3  4  5  6  7  8  9  10  Total  Rescaled
Your Score:  
Possible: 10  10  10  10  10  10  10  10  10  10  100  20

[1] (a) Let \( a_k = 2k + 1 \) for \( k \geq 0 \). Write the first five terms of the sequence.
(b) Let \( b_k = (k - 1)^3 + k + 2 \) for \( k \geq 0 \). Write the first five terms of the sequence.

[2] Find an explicit formula for the sequence that begins \( 0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \ldots \)

[3] Compute the following sums and products.
(a) \( \sum_{k=0}^{4} \frac{1}{3^k} \)  
(b) \( \prod_{k=3}^{5} (-1)^k k^3 \)  
(c) \( \prod_{k=0}^{1000} (-1)^k \)  
(d) \( \sum_{k=1}^{1000} \left( \frac{1}{k} - \frac{1}{k+1} \right) \) (hint: telescoping sum)

[4] Compute the following factorials.
(a) \( \frac{5!}{7!} \)  
(b) \( \frac{5!}{0!} \)  
(c) \( \frac{n!}{(n-2)!} \)

[5] Rewrite the following using summation notation. (Do not compute the numbers!!)
(a) \( 2 + 4 + 6 + \cdots + 738 \)  
(b) \( 2 + 4 + 6 + \cdots + 2n \)  
(c) \( 1^3 + 2^3 + 3^3 + \cdots + 597^3 \)  
(d) \( 1^3 + 2^3 + 3^3 + \cdots + n^3 \)  
(e) \( 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + 615 \cdot 616 \)  
(f) \( 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + (n-1) \cdot n \)

[6] Use repeated division by 2 to convert (by hand) the integer 205 from base 10 to base 2. Show the details.

[7] The goal is to use the Principle of Mathematical Induction to prove statement \( S \):

Statement \( S \): Any postage of at least 12 cents can be obtained by using 3 cent and 7 cent stamps.
Let \( P(n) \) be the predicate “A postage of \( n \) cents can be obtained using 3 cent and 7 cent stamps.”
Let the domain \( D \) be the set \( D = \{ n \in \mathbb{Z} \text{ such that } n \geq 12 \} \)
Using these definitions of \( P(n) \) and \( D \), statement \( S \) can be abbreviated:

\[ \text{Statement } S: \forall n \in D(P(n)) \]

Following the procedure in the Handout on Induction, prove statement \( S \).
[8] Let $P(n)$ be the predicate

$$1 + 6 + 11 + 16 + \cdots + (5n - 4) = \frac{n(5n - 3)}{2}$$

(a) Write $P(1)$. Is $P(1)$ true?

(b) Write $P(k)$.

(c) Write $P(k + 1)$.

(d) In a proof by mathematical induction that the predicate $P(n)$ is true for all $n \geq 1$, what must be shown in the inductive step?

[9] Let the domain $D$ be the set $D = \{n \in \mathbb{Z} \text{ such that } n \geq 1\}$

Following the procedure in the Handout on Induction, prove statement $S$:

$$Statement \ S: \forall n \in D \left(1 + 6 + 11 + 16 + \cdots + (5n - 4) = \frac{n(5n - 3)}{2}\right)$$

[10] (a) Let $P(n)$ be the predicate

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n + 1)}{2}\right]^2$$

(b) Write $P(1)$. Is $P(1)$ true?

(c) Write $P(k)$.

(d) Write $P(k + 1)$.

(e) In a proof by mathematical induction that the predicate $P(n)$ is true for all $n \geq 1$, what must be shown in the inductive step?

(f) Let the domain $D$ be the set $D = \{n \in \mathbb{Z} \text{ such that } n \geq 1\}$

Following the procedure in the Handout on Induction, prove statement $S$:

$$Statement \ S: \forall n \in D \left(1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n + 1)}{2}\right]^2\right)$$