One of these five solutions is correct. Circle it.
Each of the other solutions has at least one invalid step. Cross out all the invalid equal signs.

\[ \int \frac{2}{7x^5} \, dx = \int 2(7x^{-5}) \, dx = 2 \left( \frac{7x^{-4}}{-4} \right) + C = -\frac{7x^{-4}}{2} + C = -\frac{7}{2(x^4)} + C = \frac{7}{2x^4} + C \] wrong final answer

\[ \int \frac{2}{7x^5} \, dx = \int 2x^{-5} \, dx = \frac{2x^{-4}}{7(-4)} + C \times \frac{1}{7(-4)(2x^4)} + C = \frac{1}{56x^4} + C \]

\[ \int \frac{2}{7x^5} \, dx = \int 2x^{-5} \, dx = \frac{2x^{-4}}{7(-4)} + C = -\frac{x^{-4}}{7(2)} + C = \frac{1}{14x^4} + C \] correct final answer

\[ \int \frac{2}{7x^5} \, dx \neq \int 2(7x^{-5}) \, dx = 2 \left( \frac{7x^{-4}}{-4} \right) + C = \frac{7x^{-4}}{2} + C \neq \frac{1}{2x^4} + C \]

\[ \int \frac{2}{7x^5} \, dx = \int 2x^{-5} \, dx \neq \frac{2x^{-4}}{7(-6)} + C = -\frac{x^{-6}}{7(3)} + C = -\frac{1}{21x^6} + C \]

One of these four solutions is correct. Circle it.
Each of the other solutions has at least one invalid step. Cross out all the invalid equal signs.

\[ \int x(x^2 + 1) \, dx = \frac{x^2}{2} \left( \frac{x^3}{3} + 0 \right) + C = \frac{x^5}{6} + C \] can't do indefinite integral of a product of functions

\[ \int x(x^2 + 1) \, dx = \frac{x^2}{2} \left( \frac{x^3}{3} + x \right) + C = \frac{x^5}{6} + \frac{x^3}{2} + C \]

\[ \int x(x^2 + 1) \, dx = \int x^3 + x \, dx = \frac{x^4}{4} + 1 + C \] took derivative instead of indefinite integral

\[ \int x(x^2 + 1) \, dx = \int x^3 + x \, dx = \frac{x^4}{4} + \frac{x^2}{2} + C \]
\[ \frac{d}{dx} 1 = 0 \]

\[ \int 1 \, dx = x + C \]

**Power rule for derivatives**

**Power rule for indefinite integrals**
product rule for derivatives: \[ \frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x) \]

Wrong: \[ \frac{d}{dx}(f(x)g(x)) = f'(x)g'(x) \]

Invalid way of finding derivative

Product Rule for Indefinite Integrals

\[ \int f(x)g(x)\,dx = \int f(x)\,dx \cdot \int g(x)\,dx \]

Invalid way of doing indefinite integral

Product Rule for Indefinite Integrals

There is not one!!
\[ \frac{x^2}{2} \rightarrow \text{derivative} \rightarrow x \rightarrow \text{derivative} \rightarrow 1 \]

\[ \frac{d}{dx} x = 1 \quad \text{power rule for derivatives} \]

\[ \int x \, dx = \frac{x^2}{2} + C \quad \text{power rule for indefinite integrals} \]
More Difficult problems

Example #1 (Similar to 5.1 #49) Find \( \int \frac{1-x^2}{3x} \, dx \)

Solution: Notice: integrand is \( s(x) = \frac{1-x^2}{3x} \) a quotient.

There is no quotient rule for indefinite integrals.

Must rewrite so that it fits our basic rules.

\[
\begin{align*}
\frac{f(x)}{g(x)} &= \frac{1-x^2}{3x} \\
&= \frac{1}{3x} - \frac{x^2}{3x} \\
&= \left( \frac{1}{3} \right) \cdot \frac{1}{x} - \left( \frac{1}{3} \right) \cdot x
\end{align*}
\]

\[
\begin{align*}
\int s(x) \, dx &= \int \left( \frac{1}{3} \right) \cdot \frac{1}{x} - \left( \frac{1}{3} \right) x \, dx \\
&= \left( \frac{1}{3} \right) \int \frac{1}{x} \, dx - \frac{1}{3} \int x \, dx
\end{align*}
\]

\[
\begin{align*}
&= \frac{1}{3} \ln |x| - \frac{1}{3} \left( \frac{x^2}{2} \right) + C \\
&= \frac{1}{3} \ln |x| - \frac{x^2}{6} + C
\end{align*}
\]
Example 2 (Different Language)

Find the antiderivative of the derivative $C'(x) = 12x^2 - 10x$.

Solution: What this means is, find the function $C(x)$ that works in this diagram.

Unknown function $G(x)$ 

take derivative 

$C'(x) = 12x^2 - 10x$

find indefinite integral

$C(x) = \int C'(x)\,dx = \int (12x^2 - 10x)\,dx = 12\int x^2\,dx - 10\int x\,dx + K$

$= 12 \cdot \frac{x^{2+1}}{2+1} - 10 \cdot \frac{x^{1+1}}{1+1} + K$

$= \frac{12x^3}{3} - \frac{10x^2}{2} + K$

$= 4x^3 - 5x^2 + K$
Example #3 (Similar to Suggested Exercise 5.1 #55)

Find the particular antiderivative of $C'(x) = 12x^2 - 10x$ that satisfies $C(10) = 3000$.

Solution

The collection of all possible antiderivatives of $C'(x) = 12x^2 - 10x$ is what we found in Example #2. $C(x) = 4x^3 - 5x^2 + k$

Satisfy the extra condition $C(10) = 3000$

$3000 = C(10)$

$3000 = 4(10)^3 - 5(10)^2 + k$

$= 4000 - 500 + k$

$= 3500 + k$
therefore, must have \( k = -500 \), so

**Conclusion**

\[ C(x) = 4x^3 - 5x^2 - 500 \]

this is the particular antiderivative

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End of Lecture