Sign in
Pick up graded work
  - Quiz 7 solutions are printed on back
  - Current scores & grade are on Blackboard

Friday (March 24) is Exam 3
  - Bring your ID

Continuing example from yesterday

\[ f(x) = 20 - 4x - \frac{250}{x^2} \]
Find all absolute extremes on the interval \((0, \infty)\)

Find critical numbers for the function.

Strategy: Find partition numbers for \(f'(x)\)
Then find critical numbers for \(f(x)\).
To find partition numbers for \( f'(x) \)

Need to find \( f'(x) \)

Rewrite \( f(x) \) in more convenient form

\[
\begin{align*}
\text{rewrite } f(x) & = 20 - 4x - \frac{250}{x^2} \\
\text{rewrite } f(x) & = 20 - 4x - 250x^{-2}
\end{align*}
\]

\[
\text{derivative } f'(x) = \frac{d}{dx} \left(20 - 4x - 250x^{-2}\right) = \left(\frac{d}{dx}20\right) - 4\left(\frac{d}{dx}x\right) - 250\left(\frac{d}{dx}x^{-2}\right)
\]

\[
= (0) - 4(1) - 250(-2)x^{-3-1} = -4 + 500x^{-3}
\]

\[
= -4 + 500 \cdot \left(\frac{1}{x^3}\right)
\]

\[
f'(x) = -4 + \frac{500}{x^3}
\]
Partition numbers for \( f'(x) = -4 + \frac{500}{x^3} \)

- Notice \( f'(0) \) DNE so \( x=0 \) is not a partition number.
- Look for \( x \)-values that cause \( f'(x) = 0 \)

\[
0 = -4 + \frac{500}{x^3}
\]

\[
4 = \frac{500}{x^3}
\]

\[
4x^3 = 500 \\
x^3 = \frac{500}{4} = 125
\]

\[
x = 5
\]

\[
f'(5) = 0 \quad \text{so} \quad x = 5 \text{ is a partition number for } f(x)
\]
Find the critical numbers for \( f(x) \)

Observe \( f(0) = 20 - 4(0) - \frac{250}{(0)^2} \) DNE

but \( f(5) = 20 - 4(5) - \frac{250}{5^2} \) this does exist

So \( x = 5 \) is the only critical number for \( f(x) \).

If there is an absolute max or min in \( f(x) \),

it can only occur at \( x = 5 \).

Study sign behavior of \( f'(x) \) using a sign chart for \( f'(x) \)
Sign chart for $f'(x)$

- $f'(x)$ negative
- $f'(x)$ DNE
- $f'(x)$ positive
- $f'(x)$ equals 0
- $f'(x)$ negative

Sample $x = -1$

$x = 0$
- Partition number for $f'$
- Not critical number for $f$

Sample $x = 1$

$x = 5$
- Partition number for $f'$
- Critical number for $f$

Sample $x = 10$

$f'(-1) = -4 + \frac{500}{(-1)^3} = -4 + \frac{500}{-1} = -4 - 500 = \text{neg}$

$f'(1) = -4 + \frac{500}{(1)^3} = -4 + \frac{500}{1} = -4 + 500 = \text{pos}$

$f'(10) = -4 + \frac{500}{(10)^3} = -4 + \frac{500}{1000} = -4 + \frac{1}{2} = \text{neg}$
Notice: $f$ decreasing on $(-\infty, 0)$ because $f'$ neg
then $f$ increasing on $(0, 5)$ because $f'$ pos
$f$ decreasing on $(5, \infty)$ because $f'$ neg.

$f$ has a max at $x=5$ (local max)
No min at $x=0$ because $x=0$ is not a critical number.
But in fact we can see that the behavior of $f$ on the interval $(0, \infty)$ will mean that there is an **absolute max** at $x=5$.

(Absolute max for the interval $x=5$)

The absolute max is the $y$-value

\[
f(5) = 20 - 4(5) - \frac{250}{(5)^2}
\]

\[
= 20 - 20 - \frac{250}{25}
\]

\[
= -10
\]

Absolute max for the interval $(0, \infty)$ is $y = -10$ (it occurs at $x=5$)

There is no absolute min on the interval $(0, \infty)$
Section 4.6 Optimization

Optimization problems are Absolute Max/min problems

Possible complications

- May be presented as word problems.
- The function is usually not given. You have to figure out the function.
- You have to figure out the domain.
- Once you figure out the domain, you may see that it is not a closed interval.
- There may be more than one variable.
Example (similar to suggested exercises 4.6 #9, 17)

Find positive numbers \( x, y \) such that

- the sum \( 2x + y = 900 \)
- the product is maximized

Solution

Name the two equations

Equation 1 Sum: \( 2x + y = 900 \)

Equation 2 Product \( xy = A \)

Goal: find \( x, y \) that maximize the value of \( A \).

Use 1st equation to eliminate one of the variables.

Solve equation 1 for \( y \) in terms of \( x \)

\[ 2x + y = 900 \]
\[ y = 900 - 2x \]  new equation 1
Substitute Equation 1 into Equation 2

Equation 2

\[ A = xy \]
\[ = X(900-2x) \]
\[ = X \cdot 2(450-x) \]
\[ A = 2X(450-x) \]

Factored form

\[ A = 900X - 2X^2 \]

Standard form

This is an equation involving \( A + X \).
It is solved for \( A \) in terms of \( X \).
So it could be thought of as \( "A" \) as a function of \( X \).

\[ A(X) = 900X - 2X^2 = 2X(450-X) \]
What is the domain of the function \( A(x) = 900x - 2x^2 \)?

We are told that \( x > 0 \)

But we are also told that \( y > 0 \)

we know \( y = 900 - 2x \)

Must have \( x < 450 \) for \( y \) to be \( > 0 \).

So domain is \( 0 < x < 450 \)
Goal: Maximize the function $A(x) = 900x - 2x^2$

on the interval $0 < x < 450$

Find critical numbers.

Find $A'(x)$

Set $A'(x) = 0$

Solve for $x$

$A'(x) = 900(1) - 2(2x)$

$A'(x) = 900 - 4x$

$0 = A'(x) = 900 - 4x$

$4x = 900$

$x = 225$

This corresponds to the peak of the upside down parabola.

So must be a max at $x = 225$.

The corresponding $y$-value is $y = 900(225) - 2(225)^2 = 900 - 450 = 450$.
Conclusion

The best values are \( X = 225, \ y = 450 \).

The resulting product is

\[
X \cdot y = \frac{225}{450} = \text{by number}
\]