Day 22 (Wed Feb 15, 2017)

- Pick up your graded work
- Sit in Rows A-G (no body above the railing)
- Sit in groups of 2 or 3 and start Class Drill II
- Sign In
- Quiz 5 this coming Friday, Feb 17
- Exam 2 next Friday, Feb 24

Work on Class Drill II
Class Drill 11: Don’t Forget the Easy Derivative Rules

[1] Let \( f(x) = 7(x^2 + 3x + 5) \)
(A) Find \( f'(x) \), using the Product Rule to deal with the 7 in front.

\[
\begin{align*}
  f'(x) &= \left( \frac{d}{dx}(7) \right) \cdot (x^2 + 3x + 5) + 7 \cdot \frac{d}{dx}(x^2 + 3x + 5) \\
  &= 0 + 7(2x + 3) \\
  &= 7(2x + 3)
\end{align*}
\]

(B) Start over. Find \( f'(x) \) again, this time using the Constant Multiple Rule to deal with the 7 in front.

\[
\begin{align*}
  f'(x) &= 7 \cdot \frac{d}{dx}(x^2 + 3x + 5) \\
  &= 7(2x + 3)
\end{align*}
\]

Moral: If a product has one term that is a constant, you don’t need the product rule to get the derivative.

[2] Let \( f(x) = \frac{x^2 + 3x + 5}{7} \)
(A) Find \( f'(x) \), using the Quotient Rule to deal with the fraction.

\[
\frac{d}{dx} \left( \frac{x^2 + 3x + 5}{7} \right) = \frac{(x^2 + 3x + 5) \cdot (7) - (x^2 + 3x + 5) \cdot (7)}{7^2}
\]

\[
= \frac{(x^2 + 3x + 5) \cdot 7 - (x^2 + 3x + 5) \cdot 7}{7^2}
\]

\[
= \frac{(x^2 + 3x + 5) \cdot 7}{7^2}
\]

\[
= \frac{(x^2 + 3x + 5)}{7}
\]

\[
\text{cancel!!}
\]

(B) Start over. Find \( f'(x) \) again, but this time do not use the Quotient Rule. Instead, start by rewriting \( f \) as a constant times a term in parentheses. Then use the Constant Multiple rule.

\[
\begin{align*}
  f'(x) &= \left( \frac{1}{7} \right) \frac{d}{dx}(x^2 + 3x + 5) \\
  &= \left( \frac{1}{7} \right) (2x + 3)
\end{align*}
\]

Moral: If a quotient has a constant in denominator, the quotient rule is not necessary.
[3] Let \( f(x) = \frac{7}{x^3} \)
(A) Find \( f'(x) \), using the Quotient Rule to deal with the fraction. Simplify your answer.

\[
\begin{align*}
 f'(x) &= \frac{(7)' x^3 - 7(x^3)'}{(x^3)^2} \\
 &= \frac{0 x^3 - 7(3x^2)}{(x^3)^2} \\
 &= \frac{-21x^2}{(x^3)^2} = \frac{-21x^2}{x^6} = \frac{-21}{x^4}
\end{align*}
\]

(B) Start over. Find \( f'(x) \) again, but this time do not use the Quotient Rule. Instead, start by rewriting \( f \) as a constant times a power function with a negative exponent. Then use the Constant Multiple rule and the Power Rule. Simplify your answer.

\[
\begin{align*}
 \text{rewrite } f(x) &= \frac{7}{x^3} = 7 \cdot \frac{1}{x^3} = 7x^{-3} \\
 \text{derivative } f'(x) &= 7 \cdot \frac{d}{dx} x^{-3} \\
 &= 7(-3)x^{-3-1} \\
 &= -21x^{-4} = \frac{-21}{x^4}
\end{align*}
\]

[4] Let \( f(x) = \frac{7}{e^{5x}} \)
(A) Find \( f'(x) \), using the Quotient Rule to deal with the fraction. Simplify your answer.

\[
\begin{align*}
 f'(x) &= \frac{(7)'(e^{5x}) - 7(e^{5x})'}{(e^{5x})^2} \\
 &= \frac{0(e^{5x}) - 7(5e^{5x})}{(e^{5x})^2} \\
 &= \frac{-35e^{5x}}{(e^{5x})^2} = \frac{-35}{e^{5x}}
\end{align*}
\]

(B) Start over. Find \( f'(x) \) again, but this time do not use the Quotient Rule. Instead, start by rewriting \( f \) as a constant times an exponential function with a negative sign in the exponent. Then use the Constant Multiple rule and Exponential Function Rule #2. Simplify your answer.

\[
\begin{align*}
 \text{rewrite } f(x) &= \frac{7}{e^{5x}} = 7e^{-5x} \\
 \text{derivative } f'(x) &= 7 \cdot \frac{d}{dx} e^{5x} = 7(-5)e^{-5x} = -35e^{-5x} \\
 &= \frac{-35}{e^{5x}}
\end{align*}
\]
Example: \( f(x) = \frac{x^4 + y}{x^4} \) (Similar to suggested exercise 3.3#73)

Find \( f'(x) \) two ways:  
(a) using quotient rule 
(b) simplifying \( f(x) \) first, then finding \( f'(x) \)

Solution:

\[
\begin{align*}
\frac{d}{dx} f(x) &= \frac{(x^4 + y) \cdot x' - (x^4 + y) \cdot (x^4)'}{(x^4)^2} \\
&= \frac{(x^4 + y) \cdot 1 - (x^4 + y) \cdot (4x)}{(x^4)^2} \\
&= \frac{4x(x^4 + y) - (x^4 + y)(4x)}{(x^4)^2} \\
&= \frac{4x^5 - (4x^5 + 16x^3)}{(x^4)^2} \\
&= \frac{-16x^3}{x^8} \\
&= -\frac{16}{x^5}
\end{align*}
\]

\((a^b)^c = a^{b\cdot c}\)
(b) Simplify \( f(x) \) first

\[
f(x) = \frac{x^y \cdot y}{x^y} = \frac{x^y}{x^y} + \frac{y}{x^y} = 1 + y x^{-y}
\]

Now find derivative

\[
f'(x) = \frac{d}{dx} \left( 1 + y x^{-y} \right)
\]

\[
= \frac{d}{dx} 1 + y \frac{d}{dx} x^{-y}
\]

\[
= 0 + y \left( -y \right) x^{-y-1}
\]

\[
= 0 + 4 \cdot (-4) x^{-5}
\]

\[
= -16 x^{-5}
\]

\[
= -\frac{16}{x^5}
\]
Example (Similar to suggested exercise 3.3 #87)

Let \( f(x) = \frac{3x^5 - 2x^7}{\sqrt[5]{x^3}} \). Find \( f'(x) \).

Solution: eliminate the radical \( f(x) = \frac{3x^5 - 2x^7}{x^{3/5}} \).

Quotient rule would be unbearably hard.

Instead, simplify \( f(x) \) first, then find \( f'(x) \).

\[
\begin{align*}
  f(x) &= \frac{3x^5 - 2x^7}{x^{3/5}} \\
  &= 3x^{5 - \frac{3}{5}} - 2x^{7 - \frac{3}{5}} \\
  &= 3x^{\frac{22}{5}} - 2x^{\frac{32}{5}} \\
  f'(x) &= 3 \frac{d}{dx}x^{\frac{22}{5}} - 2 \frac{d}{dx}x^{\frac{32}{5}} \\
  &= \ldots \text{you can do this from here}
\end{align*}
\]

This is a trick problem sort of\ldots
Example: (Similar to suggested exercise 3.3 #65) HARD!!

Let \( f(x) = \frac{2x}{3^x} \)

Find equation of the line tangent to graph of \( f \) at \( x = 3 \)

Solution: We need to build \((y - f(a)) = f'(a)(x - a)\)

Get parts:

\( a = 3 \) \( x \)-coord of point of tangency

\( f(a) = f(3) = \frac{2 \cdot 3}{3^3} = \frac{2}{3^2} = \frac{2}{9} \) \( y \)-coord of point of tangency.

Need \( f'(a) \)

Strategy: get \( f'(x) \)

then substitute \( x = a = 3 \) in order to get \( f'(a) \)

(Friday)

End of Lecture