Continuing Section 3.2 Derivatives of Exponential Functions.

Example

An investment of \( P \) dollars earns continuously-compounded interest with rate \( r \).

(a) What is the balance at time \( t \) years?

Solution: Balance = \( A = Pe^{rt} \)

(b) What is the rate of change of the balance at time \( t \) years?

Solution: Thinking of balance \( A \) as a function of the variable \( t \). In function notation:

\[ A(t) = Pe^{rt} \]
We are being asked to find the derivative \( A'(t) \)

\[
A'(t) = \frac{d}{dt} A(t) = \frac{d}{dt} Pe^{rt} = P \frac{d}{dt} e^{rt} = P \cdot r \cdot e^{rt}
\]

Notice \( A'(t) = P \cdot r e^{rt} = r \cdot \overbrace{Pe^{rt}}^{\text{this is just } A(t)} = r \cdot A(t) \)

\[
A'(t) = r \cdot A(t) \,
\]

Special property of exponential functions
Example: Deposit $1000 into account earning 2% interest compounded continuously.

(a) Find the balance after 5 years. \( P=1000, r=0.02, t=5 \)

Solution: \( A(5) = 1000 \cdot e^{(0.02)(5)} \)

\[ A(5) = 1000 \cdot e^{1} \quad \text{exact answer} \]

\[ \approx 1105.12 \quad \text{approximate} \]

(b) Find the rate of change of the balance at time \( t = 5 \) years.

Solution: We are being asked to find \( A'(5) \)

\[ A'(5) = r \cdot A(5) = (0.02) \cdot (1000 \cdot e^{1}) = 20 \cdot e^{1} \quad \text{exact answer} \]

\[ \approx 22.10 \quad \text{dollars per year} \]
(c) Illustrate answers to (a), (b) on a graph.

Graph: 
\[ A(t) = P e^{rt} \]
\[ A(t) = 1000 e^{0.02t} \]

\[ (0, 1000) \]
\[ (5, 1105) \]

\[ t = 5 \text{ years} \]

\[ m = 22^{10} \text{ dollars per year} \]