Day 52 (Mon Nov 28, 2016)

- Pick up graded Exams
- Sit in Groups of 2 or 3
- Quiz 10 in Class this Wednesday, Nov 30
- Final Exam a week from today.
  - Monday Dec 5, 10:10am - 12:10pm
  - Notice start time!!
  - Bring your OU ID (sorry!)
  - I’ll post study guide later today

Today: Continuing Section 6.1
Area Between Curves
More Difficult Examples + Total Change Problems
Example #1 (Similar to Suggested Exercise C.1 #67)

Find the area of the region bounded by

\[ y = x^5 \quad \text{and} \quad y = 16x \]

notice: interval is not given
not told whether graphs cross
not told which graph is on top

<table>
<thead>
<tr>
<th>x</th>
<th>y = x^5</th>
<th>y = 16x</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>(-3)^5 = -243</td>
<td>16(-3) = -48</td>
</tr>
<tr>
<td>-2</td>
<td>(-2)^5 = -32</td>
<td>16(-2) = -32</td>
</tr>
<tr>
<td>-1</td>
<td>(-1)^5 = -1</td>
<td>16(-1) = -16</td>
</tr>
<tr>
<td>0</td>
<td>0^5 = 0</td>
<td>16(0) = 0</td>
</tr>
<tr>
<td>1</td>
<td>1^5 = 1</td>
<td>16(1) = 16</td>
</tr>
<tr>
<td>2</td>
<td>2^5 = 32</td>
<td>16(2) = 32</td>
</tr>
<tr>
<td>3</td>
<td>3^5 = 243</td>
<td>16(3) = 48</td>
</tr>
</tbody>
</table>

\[ y = 16x \]
\[ (2, 32) \]
What interval should we use as the end of our region?
The only regions bounded by the graphs are from \( x = -2 \) to \( x = 2 \).

We could compute \( USA = \)

\[
\int_{x=-2}^{x=0} x^5 - 16x \, dx + \int_{x=0}^{x=2} 16x - x^5 \, dx
\]

But the two blue regions have exactly the same area, so we could compute just one integral and then multiply by 2 to get \( USA \).

\[
USA = 2 \times \int_{x=0}^{x=2} 16x - x^5 \, dx
\]
\[ \text{USA} = 2 \cdot \left[ \int_{x=0}^{x=2} 16x - x^5 \, dx \right] \]

\[ = 2 \cdot \left[ \left( \frac{16x^{1+1}}{1+1} - \frac{x^{5+1}}{5+1} \right) \right]_{x=0}^{x=2} \]

\[ = 2 \cdot \left( \frac{16x^2}{2} - \frac{x^6}{6} \right) \bigg|_{x=0}^{x=2} \]

\[ = 2 \cdot \left( 8x^2 - \frac{x^6}{6} \right) \bigg|_{x=0}^{x=2} \]

\[ = 2 \cdot \left( 8(2)^2 - \frac{(2)^6}{6} \right) - \left( 8(0)^2 - \frac{(0)^6}{6} \right) \]

\[ = 2 \cdot \left[ 8(4) - \frac{64}{6} \right] = 2 \left[ 32 - \frac{32}{3} \right] = 2 \left[ 32 \left( \frac{2}{3} \right) \right] = \frac{32(4)}{3} \]
Example #2 Find area between graphs of \( y = e^{x} \) and \( y = -\frac{1}{x} \) for \( 1 \leq x \leq 3 \) 

(Similar to Suggested exercise 6.1 #5-7)

Notice: no interval is given. Good

- We're not told anything about the graphs

Solution: Make graphs.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = e^{x} )</th>
<th>( y = -\frac{1}{x} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = 1 )</td>
<td>( e^{(1)} = e \approx 2.7 )</td>
<td>(-\frac{1}{1} = -1 )</td>
</tr>
<tr>
<td>( x = 2 )</td>
<td>( e^{2} )</td>
<td>(-\frac{1}{2} )</td>
</tr>
<tr>
<td>( x = 3 )</td>
<td>( e^{3} )</td>
<td>(-\frac{1}{3} )</td>
</tr>
</tbody>
</table>

So \( y = e^{x} \) is the top graph on whole interval.
So \( \text{USA} = \int_{x=1}^{x=3} e^x - \left( \frac{-1}{x} \right) \, dx = \int_{x=1}^{x=3} e^x + \frac{1}{x} \, dx \)

\[
= \left[ e^x + \ln(x) \right]_{x=1}^{x=3}
\]

\[
= (e^3 + \ln(3)) - (e^{1} + \ln(1))
\]

\[
= e^3 + \ln(3) - e
\]

\( \approx 18.5 \)
For tomorrow

Similar to 6.1 #95

Yeast culture growing at a rate $W(t) = 0.5e^{0.35t}$ gm/hour

(A) Find area between graph of $W(t)$ and the t-axis over the interval $[0, 4]$

(B) Important: Interpret the results of (A).

What does (A) tell us??

Well start with this example tomorrow (Tuesday)

[End of Lecture]