• Pick up Graded Work
• Quiz 6 Today
• Quiz 7 Next Friday
• Exam #3 two weeks from today

Today: A few more examples on Section 4.1 Material
Start by finishing example from Wednesday

For \( f(x) = -x^4 + 50x^2 + 7 \)
We find \( f'(x) = -4x^3 + 100x = -4(x+5)(x)(x-5) \)

(A) Partition numbers for \( f'(x) \) are \( x = -5, x = 0, x = 5 \) they cause \( f' = 0 \)
(B) Critical numbers for \( f \) are the same three numbers.
(C) Find the intervals where \( f \) is increasing and intervals where \( f \) is decreasing
Strategy: Study sign of \( f' \) with a sign chart.
Sign Chart for \( f'(x) = -4(x+5)(x)(x-5) \)

<table>
<thead>
<tr>
<th>( f' ) pos</th>
<th>( f' = 0 )</th>
<th>( f' ) neg</th>
<th>( f' = 0 )</th>
<th>( f' ) pos</th>
<th>( f' = 0 )</th>
<th>( f' ) neg</th>
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<tbody>
<tr>
<td>( X = -5 )</td>
<td>( X = 0 )</td>
<td>( X = 5 )</td>
<td>( X = 6 )</td>
<td></td>
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</table>

Sample \( X = -6 \)

\[
f'(-6) = -4(-6+5)(-6)(-6-5) = (-)(-)(-) = pos
\]

Sample \( X = -1 \)

\[
f'(-1) = -4(-1+5)(-1)(-1-5) = (+)(-)(-) = neg
\]

Sample \( X = 1 \)

\[
f'(1) = -4(1+5)(1)(1-5) = (+)(+)(-) = pos
\]

Sample \( X = 6 \)

\[
f'(6) = -4(6+5)(6)(6-5) = (+)(+)(+) = neg
\]

(2) \( f \) increasing on the intervals \((-\infty, -5) \) and \((0, 5)\) because \( f' \) is positive.

\( f \) decreasing on the intervals \((-5, 0) \) and \((5, \infty)\) because \( f' \) is negative.
D) Find local maxs + local mins

The locations of the max + mins are at $x = -5, 0, 5$ because those pass the 1st derivative test.

- Partition numbers for $f'$ ✓
- Critical numbers for $f$ ✓
- $f$ continuous (polynomial) ✓
- $f'$ changes sign ✓

The max + mins are the y-values on graph of $f(x) = -x^4 + 50x^2 + 7$.

$f(-5) = -(-5)^4 + 50(-5)^2 + 7 = -625 + 50(25) + 7 = -625 + 1250 + 7 = 625 + 7 = 632$

$f(0) = -(0)^4 + 50(0)^2 + 7 = 7$

$f(5) = -5^4 + 50(5)^2 + 7 = 632$
Conclusion

Local max $y = 632$ occurs at $x = -5$ because $f'$ changes from pos to zero to neg there.

Local min $y = 7$ occurs at $x = 0$ because $f'$ changes from neg to zero to pos.

Local max $y = 632$ occurs at $x = 5$ because $f'$ changes from pos to zero to neg.
Another Example: Let \( f(x) = xe^{(-x)} \).

Same questions as on previous example:
- Partition numbers for \( f' \)?
- Critical numbers for \( f \)?
- Where is \( f \) incr / decr?
- Local max / mins?

Solution:
(A) \[ f'(x) = \frac{d}{dx}(xe^{(-x)}) \]

\[ = \left(\frac{d}{dx}x\right)e^{(-x)} + x\left(\frac{d}{dx}e^{(-x)}\right) \]

\[ = (1)e^{(-x)} + x\left((-1)e^{(-x)}\right) \]

\[ = (1 - x)e^{(-x)} \]

Product rule
\[ \frac{de^{kx}}{dx} = ke^{(kx)} \]
Partition numbers for $f'$??

$$f'(x) = (1-x)e^{(-x)}$$

Domain: all real numbers

No $x$-values where $f'(x)$ does not exist.

Find $x$-values where $f'(x) = 0$

$$f'(x) = (1-x)e^{(-x)} = 0$$

$e^{anything} > 0$ so $e^{-x}$ will never be zero!

So this term must be zero

So the only solution is $x = 1$
So only partition number for $f'(x)$ is $x=1$.

(B) Find critical numbers for $f$.

Since $f(x) = xe^{\sin x}$ is continuous, we know $f(x)$ always exists, so $f'(1) = 1 \cdot e^{\sin 1}$ exists.

So $x=1$ is also a CRITICAL NUMBER for $f$.

We will resume this example on Monday.

End of Lecture