• Put your phone away.
• Put your phone away.
• Don't make me say it again.
• Pick up your graded Quizzer
• Sign In sheet will come around in awhile
• Exam I on Friday
  • Bring your on ID.
• Exam information on Course Web Page
Trick Problems

Example similar to 2.5 # 8

Let \( f(x) = \frac{2x^5 - 4x^3 + 2x}{x^3} \). Find \( f'(x) \)

Solution: We will use the Derivative Rules from Section 2.5.

This is a quotient, and we don't (yet) know any derivative rules for quotients.

(Even when we do later learn the "Quotient Rule" for derivatives, it will not be the smart way to solve this problem.)
Step 1: Rewrite \( f(x) \) as a sum of terms involving constants times power functions.

\[
f(x) = \frac{2x^5 - 4x^3 + 2x}{x^3} = \frac{2x^5}{x^3} - \frac{4x^3}{x^3} + \frac{2x}{x^3}
\]

\[
= 2x^2 - 4 + \frac{2}{x^2}
\]

\[
= 2x^2 - 4 + 2x^{-2}
\]

Step 2: Find the derivative

\[
f'(x) = \frac{d}{dx} \left( 2x^2 - 4 + 2x^{-2} \right)
\]

\[
= 2 \frac{d}{dx} x^2 - 4 \frac{d}{dx} (1) + 2 \frac{d}{dx} x^{-2}
\]

\[
= 2 \frac{d}{dx} x^2 - 4 \frac{d}{dx} (1) + 2 \frac{d}{dx} x^{-2}
\]

Sum Rule + Constant Multiple Rule
\[\frac{d}{dx}(2x^{-2}) - 4(0) + 2(-2x^{-2-1})\]
\[= 4x' - 0 - 4x^{-3}\]
\[= 4x - \frac{4}{x^3}\]

power rule

rewrite without negative exponent
Another leftover from Section 2.5:

**The Tangent Line Equation**

Review **Point Slope Form** for the equation of the line that passes through
- the known point \((a, b)\)
- with known slope \(m\)

\[(y - b) = m(x - a)\]

Review **Things that we know about the line tangent to graph of \(f\) at \(x = a\)**
- The line touches the graph at the point \((a, f(a))\) (the point of tangency)
- The line has slope \(m = f'(a)\)
So we can build the point slope form of the equation for the line tangent to graph of $f$ at $x=a$

$$(y - f(a)) = f'(a) \cdot (x - a)$$

Example: Let $f(x) = x^3 - 9x^2 + 15x + 25$

Find equation of the line tangent to graph of $f$ at $x=2$.

Convert your equation to slope intercept form.
Solution

We need to build \((y - f(a)) = f'(a)(\frac{y - x}{a - x})\)

Get parts

\[a = 2\]
\[x\)-coordinate of the point of tangency

\[f(a) = f(2) = (2)^2 - 9(2)^2 + 15(2) + 25\]
\[= 8 - 36 + 30 + 25\]
\[= 27\]
\[y\)-coordinate of point of tangency

\[f'(x) = \frac{d}{dx}(x^3 - 9x^2 + 15x + 25)\]
\[= 3x^2 - 9.2x + 15(1) + 0\]
\[= 3x^2 - 18x + 15\]

\[f'(a) = f'(2) = 3(2)^2 - 18(2) + 15 = 12 - 36 + 15 = -9\]
Assemble the parts

\[(y - 27) = (-9)(x - 2)\]

Now convert to slope intercept form

\[y - 27 = (-9)(x) + (-9)(-2)\]

\[y - 27 = -9x + 18\]

\[y = -9x + 45\]
Class Drill 8: Questions about Tangent Lines

Let \( f(x) = x^3 - 3x^2 - 9x + 11 \)

(a) Find \( f'(x) \).
Use the techniques of Section 2.5. (That is, DO NOT use the Definition of the Derivative.)
Show all details clearly and use correct notation.

(b) Find the slope of the line that is tangent to the graph of \( f \) at \( x = 3 \).

(c) Find the slope of the line that is tangent to the graph of \( f \) at \( x = 0 \).

(d) Find the \( x \)-coordinates of all points on the graph of \( f \) that have horizontal tangent lines.
(e) Find the equation of the line that is tangent to the graph of \( f \) at \( x = 2 \). Show all details clearly and present your equation in slope intercept form.

(Remember that the approach is to build the general form of the equation for the tangent line in point-slope form

\[
(y - f(a)) = f'(a) \cdot (x - a)
\]

and then convert the equation to slope-intercept form.)
Section 2.7 Marginal Analysis

Marginal Quantities

Basic definition
words: "Marginal BLAH"
meaning: "the derivative of BLAH"

Example
Suppose Revenue function is \( R(x) = 5x - 0.02x^2 \)
Find the Marginal Revenue

Solution
Marginal Revenue = \( R'(x) = \frac{d}{dx}(5x - 0.02x^2) \)
\[ \begin{align*}
&= \frac{d}{dx} 5x - 0.02 \frac{d}{dx} x^2 \\
&= 5(1) - 0.02(2x) \\
R'(x) &= 5 - 0.04x
\end{align*} \]