Continuing Section 2.2 Limits Involving Infinity

Yesterday: Limits involving Infinity for a function given by a graph.

Today: Limits involving Infinity for a function given by a formula.

Class Driv #3
Class Drill 3: Guessing Limits by Substituting in Numbers

Without using a calculator, answer the following questions about the function

\[ f(x) = \frac{x^2 - 6x + 5}{x^2 - 8x + 15} \]

Part 1: Function Values

(1) Factor \( f \). (Check your factorizations by multiplying.)

\[ f(x) = \frac{(x-1)(x-5)}{(x-3)(x-5)} \]

Check: \( (x-1)(x-5) = \)

Check this: \( x^2 - x - 5x + 5 = \)

\( = x^2 - 6x + 5 \)

(2) Are you allowed to cancel factors in the factored form of \( f \)? Explain why you think you are allowed to cancel, or why you are not.

\textit{Cannot cancel. We don't know value of \( x \), so we don't know if \( x-5 \) is zero or not.}

(3) Find \( f(1) \) by substituting \( x = 1 \) into the factored version of \( f \).

\[ f(1) = \frac{(1-1)(1-5)}{(1-3)(1-5)} = \frac{0(-4)}{3(-4)} = \frac{0}{-12} = 0 \]

(4) Find \( f(3) \) by substituting \( x = 3 \) into the factored version of \( f \).

\[ f(3) = \frac{(3-1)(3-5)}{(3-3)(3-5)} = \frac{(2)(-2)}{0(-2)} = \frac{-4}{0} \quad \text{DNE !!} \]

(5) Find \( f(5) \) by substituting \( x = 5 \) into the factored version of \( f \).

\[ f(5) = \frac{(5-1)(5-5)}{(5-3)(5-5)} = \frac{0}{0} \quad \text{DNE !!} \]

Part 2: Limits

Using the factored form of \( f \), compute the following values and guess the limits.

Guessing the limit at \( x = 5 \).

\textit{(Just leave answers as an expression ready to type into a calculator.)}

(11) \( f(5.1) = \frac{(5.1-1)(5.1-5)}{(5.1-3)(5.1-5)} = \frac{4.1}{2.1} = \text{close to 2} \)

(12) \( f(5.01) = \frac{(5.01-1)(5.01-5)}{(5.01-3)(5.01-5)} = \frac{4.01}{2.01} = \text{closer to 2} \)

(13) \( f(5.001) = \quad \text{0.000} = \frac{4.001}{2.001} = \text{really close to 2} \)

(15) \( \lim_{x \to 5^+} f(x) = 2 \)
(16) \( f(4.9) = \frac{(4.9 - 1)(4.9 - 5)}{(4.9 - 3)(4.9 - 5)} = \frac{3.9}{1.9} \) close to 2


(18) \( f(4.999) = \frac{0.00 = 3.999}{1.999} \) really close to 2

(20) Guess \( \lim_{x \to 5^-} f(x) = 2 \)

(21) Guess \( \lim_{x \to 5^+} f(x) = 2 \) because left & right limits match

**Guessing the limit at x = 3. (Simplify your answers.)**

(11) \( f(3.1) = \frac{(3.1 - 1)(3.1 - 5)}{(3.1 - 3)(3.1 - 5)} = \frac{2.1}{0.1} = 21 \)

(12) \( f(3.01) = \frac{(3.01 - 1)(3.01 - 5)}{(3.01 - 3)(3.01 - 5)} = \frac{2.01}{0.01} = 201 \)

(13) \( f(3.001) = \frac{(3.001 - 1)(3.001 - 5)}{(3.001 - 3)(3.001 - 5)} = \frac{2.001}{0.001} = 2001 \)

(15) Guess \( \lim_{x \to 3^+} f(x) = \infty \)

(16) \( f(2.9) = \)

(17) \( f(2.99) = \) similar

(18) \( f(2.999) = \)

(20) Guess \( \lim_{x \to 3^-} f(x) = -\infty \)

(21) Guess \( \lim_{x \to 3} f(x) = \text{DNE because left & right limits don't match} \)