Finishing Section 2.1
Reviewed definition of Instantaneous Rate of Change (also called the derivative of f at x=a)

\[ f'(a) = \lim_{b \to a} \frac{f(b) - f(a)}{b - a} \]

= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}

= slope M of the line tangent to graph of f at x=a
Reviewed the definition of the line tangent to graph of $f$ at $x = a$. It is the line that has these two properties:

- It touches the graph of $f$ at the point $(a, f(a))$ (the "point of tangency")
- It has slope $m = f'(a)$

Reviewed the general equation for the line tangent to graph of $f$ at $x = a$.

$$(y - f(a)) = f'(a)(x - a)$$

Worked on graph work 4:

for $f(x) = 5x^2 - 2x + 7$,

a) Find $f'(a)$ using the definition of the derivative.

b) Find the equation for the line tangent to graph of $f$ at $x = 2$.

Present your equation in slope-intercept form,
Section 2.4 The Derivative as a function

In Section 2.1, we had

\[ f'(a) = \lim_{b \to a} \frac{f(b) - f(a)}{b - a} = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} = m = \text{derivative at a particular } x\text{-value } \frac{\text{number}}{\text{number}} \]

In Section 2.4, we have

\[ f'(x) = \lim_{b \to x} \frac{f(b) - f(x)}{b - x} = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \text{the derivative function} \frac{\text{variable}}{\text{function}} \]
We've actually done derivative problems before!

Example: Group Work 3: \( \lim_{{h \to 0}} \frac{(2+h)^3 - 8}{h} \)

Is this a number or a function? A number

So this is \( f'(a) \) for some function \( f(x) \) and some number \( a \).

What is \( f(x) \)? Notice the expression in 2th

What is \( a \)? Notice the expression in 2th

The number 2 is \( a \). \( a = 2 \).

Notice \( (2+h)^2 \)

we see that \( f(\ ) \) is \( (\ )^3 \)

\[ f(\ ) = (\ )^3 \]

\[ f(x) = x^3 \]

The group work found \( f'(2) \) for \( f(x) = x^3 \)
Second Problem on Group Work 3

\[ \lim_{{h \to 0}} \frac{1}{{x+h}} - \frac{1}{{x}} \]

Very messy problem, but it does not require any trick; you just have to find common denominator.

function or number? Function of variable \( x \).

So the expression represents \( f'(x) \) for some \( f(x) \).

What is \( f(x) \)? Observe \( \frac{1}{{x}} \) in upper right.

So \( f(x) = \frac{1}{{x}} \)
Exercise 1.4 #21

\[
\lim_{{n \to 0}} \frac{\sqrt{9+n} - 3}{n}
\]

Function or number? 
Numbers!

So this is \( f'(a) \) for some \( f(x) \) and some \( a \).

What is \( a \)? \( a = 9 \) because 9th term

What is \( f(x) \)?

\[
f(c) = \sqrt{c}
\]

So \( f(x) = \sqrt{x} \)

This exercise finds \( f'(9) \) for \( f(x) = \sqrt{x} \)
Group Work 5

for $f(x) = \frac{3x + 5}{x + 7}$, find $f'(x)$ using the Definition of the Derivative.

End of Lecture