Day 2 (Wed Jan 13) (MATH 2301 Barsamian)

Monday's Class Drill 1 was about graph \( \Rightarrow \) description of limit behavior.

Today: Continuity Section 1.3 Limits
Start with example

description of limit behavior \( \Rightarrow \) graph

Example (exercise 1.3 #9)
Sketch a graph of a function \( f \) that has all five of these properties:

- \( f(3) = 5 \)
- \( f(2) = 6 \)
- \( \lim_{x \to 3^+} f(x) = 4 \)
- \( \lim_{x \to 3^-} f(x) = 2 \)
- \( \lim_{x \to 3^-} f(x) = 1 \)
Solution: The given properties tell us that five locations will be important: they are 

\( (x, y) = (3, 5) \), \( (x, y) = (2, 6) \), \( (x, y) = (3, 4) \), \( (x, y) = (3, 2) \), \( (x, y) = (2, 1) \)

Plot these locations using open circles.
Now use given info to determine what happens at those locations.

\[ f(x) \]

\[ (2, 6) \]
\[ (-2, 1) \]
\[ (3, 5) \]
\[ (3, 4) \]
\[ (3, 2) \]

So far we have considered limit examples where function \( f \) is given by a graph. Now consider analytic examples, where \( f \) is described by a formula. Class Drill 2
Details for (5), (6)

(5) \[ f(x) = \frac{x(x-3)}{(x-2)(x-3)} \]

Check!!
\[ x(x-3) = x^2 - 3x \checkmark \]
\[ (x-2)(x-3) = x^2 - 2x - 3x + 6 = x^2 - 5x + 6 \checkmark \]

(6) Can we cancel factors?

Compare these two functions

\[ f(x) = \frac{x(x-3)}{(x-2)(x-3)} \]

Domain: all \( x \neq 2, 3 \)

\( f(2) \) DNE

\( f(3) \) DNE

\[ g(x) = \frac{x}{x-2} \]

Domain: all \( x \neq 2 \)

\( g(2) \) DNE

\( g(3) = \frac{3}{3-2} = \frac{3}{1} = 3 \)

The functions are not the same! They have different domains.
MATH 2301 (Barsamian) Group Work 2: Guessing Limits by Substituting in Numbers

Without using a calculator, answer the following questions about the function.

\[ f(x) = \frac{x^2 - 3x}{x^2 - 5x + 6} \]

Part 1: Function Values

(1) Find \( f(1) \).

(2) Find \( f(2) \).

\[ = \frac{2^2 - 3(2)}{2^2 - 5(2) + 6} = \text{hard!} = \ldots = \frac{-2}{0} = \text{undefined} \]

(3) Find \( f(3) \).

\[ = \frac{3^2 - 3(3)}{3^2 - 5(3) + 6} = \text{pain} = \ldots = \frac{0}{0} = \text{undefined} \]

(4) Find \( f(3.1) \) by substituting \( x = 3.1 \) into the above expression. No calculators.

\[ = \frac{(3.1)^2 - 3(3.1)}{(3.1)^2 - 5(3.1) + 6} = \text{too hard! No way!} \]

(5) Factor \( f \). (Check your factorizations by multiplying.)

\[ f(x) = \frac{x(x-3)}{(x-2)(x-3)} \]

(6) Are you allowed to cancel factors in the factored form of \( f \)? Explain why you think you are allowed to cancel, or why you are not.

\[ \text{No! That would change the domain!} \]

(7) Find \( f(1) \) by substituting \( x = 1 \) into the factored version of \( f \).

(8) Find \( f(2) \) by substituting \( x = 2 \) into the factored version of \( f \).

\[ = \frac{2(2-3)}{(2-2)(2-3)} = \frac{2(1)}{0(1)} = \frac{2}{0} = \text{undefined!} \]

(9) Find \( f(3) \) by substituting \( x = 3 \) into the factored version of \( f \).

\[ = \frac{3(3-3)}{(3-2)(3-3)} = \frac{3 \cdot 0}{1 \cdot 0} = \frac{0}{0} = \text{undefined} \]
Part 2: Limit as $x \to 3$

Using the factored form of $f$, compute the following values and guess the limits. (No calculators)
(Simplify your expressions by cancelling when possible, but don’t bother doing the division. That is, leave your answers as fractions.)

(10) Find $f(3.1)$ by substituting $x = 3.1$ into the factored version of $f$.
$$f(3.1) = \frac{3.1(3.1-3)}{(3.1-2)(3.1-3)} = \frac{3.1}{1.1}$$

(11) $f(3.01) = \frac{(3.01)(3.01-3)}{(3.01-2)(3.01-3)} = \frac{3.01}{1.01}$

(12) $f(3.001) = \frac{3.001}{1.001}$

(13) Guess $\lim_{x \to 3^+} f(x) = 3$

(14) $f(2.9) = \frac{2.9(2.9-3)}{(2.9-2)(2.9-3)} = \frac{2.9}{0.9}$

(15) $f(2.99) = \frac{2.99(2.99-3)}{(2.99-2)(2.99-3)} = \frac{2.99}{0.99}$

(16) $f(2.999) = \frac{2.999}{0.999}$

(17) Guess $\lim_{x \to 3^-} f(x) = 3$

(18) Guess $\lim_{x \to 3} f(x) = 3$
Part 3: Limit as \( x \to 2 \)
Using the factored form of \( f \), compute the following values and guess the limits. (No calculators) (Simplify your expressions by cancelling when possible.)

(19) \( f(2.1) = \frac{2.1(2.1-3)}{(2.1-2)(2.1-3)} = \frac{2.1}{0.1} = \frac{2.1}{\frac{10}{1}} = 21 \)

(20) \( f(2.01) = \frac{2.01}{0.01} = 201 \)

(21) \( f(2.001) = \frac{2.001}{0.001} = 2001 \)

(22) Describe in words the trend that you observe in (19), (20), (21).

As \( x \) approaches 2 from the right, the y-values get more and more positive without bound.

(23) Does \( \lim_{x \to 2^+} f(x) \) exist? Explain.

No because there is no real number \( L \) that the y-values are approaching.

(24) \( f(1.9) = \frac{1.9(1.9-3)}{(1.9-2)(1.9-3)} = \frac{1.9}{-0.1} = -19 \)

(25) \( f(1.99) = \frac{1.99(1.99-3)}{(1.99-2)(1.99-3)} = \frac{1.99}{-0.01} = -199 \)

(26) \( f(1.999) = \ldots = -1999 \)

(27) Describe in words the trend that you observe in (24), (25), (26).

As \( x \) approaches 2 from the left, the y-values get more and more negative without bound.

(28) Does \( \lim_{x \to 2^-} f(x) \) exist? Explain.

No because there is no real number \( L \) that works.

(29) Does \( \lim_{x \to 2} f(x) \) exist? Explain.

No because the left and right limits don't exist.

[End of Lecture]