The First Derivative Test
MATH 2301 (Barsamian) Group Work 23

For some unknown function $f$, a sign chart for $f'$ is given below, along with some important $y$ values for $f$. Assume that $f$ is continuous everywhere on its domain.

\[ f'(7)DNE \quad f'(14) = 0 \quad f'(21) = 0 \quad f'(28) = 0 \quad f'(35)DNE \]
\[ f(7)DNE \quad f(14) = 1 \quad f(21) = 6 \quad f(28) = 4 \quad f(35) = 2 \]

\[
\begin{array}{cccccc}
  x & 7 & 14 & 21 & 28 & 35 \\
  f' & + & + & + & + & + \\
  f' & - & - & - & - & - \\
  f' & + & + & + & + & + \\
  f' & - & - & - & - & - \\
  f' & + & + & + & + & + \\
\end{array}
\]

(A) Fill in this table:

<table>
<thead>
<tr>
<th></th>
<th>$c = 7$</th>
<th>$c = 14$</th>
<th>$c = 21$</th>
<th>$c = 28$</th>
<th>$c = 35$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1: Is it true that $f''(c) = 0$ or $f''(c)$ is undefined?</td>
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<tr>
<td>Test 2: Is $f(c)$ defined?</td>
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<tr>
<td>Test 3: Is $f$ continuous at $x = c$?</td>
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<tr>
<td>Test 4: Does $f'$ change sign at $x = c$?</td>
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</tr>
</tbody>
</table>

(B) Based on your table, what are the $x$-coordinates where local extrema occur? For each one, say whether it is a local max or a local min.

(C) What are the corresponding $y$-coordinates? That is, what are the values of the local extrema?

(D) Sketch a possible graph of $f(x)$. 

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The First Derivative Test for Local Extrema

**Test 1:** $f'(c) = 0$ or $f''(c) DNE$

If the number $x = c$ passes test 1, then $c$ is called a **partition number for $f'$**.

**Test 2:** $f(c)$ exists.

**Test 3:** $f$ is continuous at $x = c$.

**Test 4:** $f'$ changes sign at $x = c$.

If the number $x = c$ passes tests 1, 2, 3, then $c$ is the **location** of a local max or min of $f$. The **value** of the local max or min is the corresponding $y$-value, $f(c)$. 

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For some unknown function $f$, a sign chart for $f'$ is given below, along with some important $y$ values for $f$. Assume that $f$ is continuous everywhere on its domain.

\[ f'(7)DNE \quad f'(14) = 0 \quad f'(21) = 0 \quad f'(28) = 0 \quad f'(35)DNE \]
\[ f(7)DNE \quad f(14) = 1 \quad f(21) = 6 \quad f(28) = 4 \quad f(35) = 2 \]

\[
\begin{array}{cccccc}
  x & 7 & 14 & 21 & 28 & 35 \\
  f' & + & + & + & + & + \\
  f' & - & - & - & - & - \\
  f' & + & + & + & + & + \\
  f' & - & - & - & - & - \\
  f' & + & + & + & + & + \\
\end{array}
\]