MATH 1350 Day 49 (Tues April 12, 2016)

- Pick up Graded Work
- Quiz 8 Solutions are printed on back of Quiz 8
- Exam 4 tomorrow in class
  - Go to bathroom before coming to the exam room.

Continuing Section 5.5 Fundamental Theorem of Calculus

One More Total Change Problem

The salvage value for a copier is changing with derivative

\[ V'(t) = 15,000(t - 9) \]

for \( 0 \leq t \leq 7 \)

(a) What is the change in value of the copier in the first two years? \( (0 \leq t \leq 2) \)
(b) In the second year after that? \( (2 \leq t \leq 4) \)
Solution: We are being asked for \( \Delta V = V(2) - V(0) \).

**Strategy:** Use Fundamental Theorem of Calculus.

\[
\Delta V = V(2) - V(0) = \int_{t=0}^{t=2} V'(t) \, dt
\]

\[
= \int_{t=0}^{t=2} 15000 \, (t - 9) \, dt
\]

\[
= 15000 \left( \frac{t^2}{2} - 9t + C \right) \bigg|_{t=0}^{t=2}
\]

\[
= 15000 \left( \frac{2^2}{2} - 9(2) + C \right) - \left( \frac{0^2}{2} - 9(0) + C \right)
\]

\[
= 15000(2 - 18)
\]

\[
= 15000(-16)
\]

\[
= -240,000
\]

**Copier lost $240,000 in value in first two years.**
(b) Change in value in two years after that

\[ \Delta V = V(4) - V(2) = \int_{t=2}^{t=4} V'(t) \, dt \]

\[ \Delta V = 15,000 \left( \frac{t^2}{2} - 9t + C \right) \bigg|_{t=2}^{t=4} \]

\[ = 15,000 \left( \left( \frac{4^2}{2} - 9(4) + C \right) - \left( \frac{2^2}{2} - 9(2) + C \right) \right) \]

\[ = 15,000 \left( 8 - 36 - 2 + 18 \right) \]

\[ = 15,000 (26 - 38) \]

\[ = 15,000 (-12) \]

\[ = -180,000 \]

Copier lost $180,000 in value in the next two years.
New Question 2

How much is the copier worth at time $t = 4$?
How much was the copier purchased for?

Solution

We are being asked to find $V(4)$ and $V(0)$.

The best we can do is

$$V(t) = \int V'(t)\,dt = \int 15,000(t-9)\,dt = 15,000 \int (t-9)\,dt =$$

$$= 15,000 \left( \frac{t^2}{2} - 9t \right) + C$$

So $V(4) = 15,000 \left( \frac{4^2}{2} - 9\cdot4 \right) + C = 15,000 (8 - 36) + C$

$$= 15,000 (-28) + C = -420,000 + C$$

We can't know $V(4)$ because we don't know the value of $C$!
Original purchase price

\[ V(0) = 15000 \left( \frac{0^2}{2} - 9(0) \right) + c = c \]

we don't know what \( C \) is.

But we see that it represents the original purchase price.
Section 5.5, continued
we have learned about:
- The Fundamental Theorem of Calculus (FTC)
- Applications of FTC to total change problems.

Another Application of Fundamental Theorem

The Average Change over

The Average Value of a Function over an Interval
Idea: Given continuous function $f(x)$ and an interval $[a, b]$

Question: How high would a rectangle sitting on the same interval $[a, b]$ need to be to have the same signed area?  

Solution: Area on left = $\int_{a}^{b} f(x) \, dx$

Area on right = base $\times$ height = $(b-a) \cdot h$
Must have \((b-a) \cdot h = \int_{x=a}^{x=b} f(x) \, dx\)

Divide both sides by \(b-a\)

\[ h = \frac{1}{b-a} \int_{x=a}^{x=b} f(x) \, dx \]

**Definition**

Words: The average value of \(f(x)\) over the interval \([a, b]\)

**Meaning:** the number \(h = \frac{1}{b-a} \int_{x=a}^{x=b} f(x) \, dx\)

**Graphical interpretation:** The number \(h\) is the height of a rectangle sitting on the interval \([a, b]\) that would enclose the same signed area as the signed area of \(f(x)\) on the same interval.
Example: Find the average value of \( g(t) = 4t + 3t^2 \)

over the interval \(-2 \leq t \leq 5\)

**Solution**

\[
h = \frac{1}{5 - (-2)} \int_{-2}^{5} (4t + 3t^2) \, dt
\]

\[
= \frac{1}{7} \left[ \frac{4t^2}{2} + \frac{3t^3}{3} + C \right] \bigg|_{t=-2}^{t=5}
\]

\[
= \frac{1}{7} \left[ 2t^2 + t^3 + C \right] \bigg|_{t=-2}^{t=5}
\]

\[
= \frac{1}{7} \left[ 2(5)^2 + 5^3 + C \right] - \left( 2(-2)^2 + (-2)^3 + C \right)
\]

\[
= \frac{1}{7} \left[ 50 + 125 - 8 + 8 \right] = \frac{175}{7} = 25 = h
\]