Continuing Section 4.5 Absolute Extrema today: What happens when the Extreme Value Theorem cannot be used.

Class Drill 21 Local and Absolute Extrema

Example Where Closed Interval Method Won't Work

Let \( f(x) = x^4 - 6x^2 + 5 \)

Find absolute extrema on the interval \((-\infty, 0)\)

Notice: Domain is not a closed interval

So we are not guaranteed any absolute extrema.

Ask first: Will there be an absolute max?

Will there be an absolute min?
The Extreme Value Theorem says that if a function \( f \) is continuous on a closed interval \([a,b]\), then \( f \) will have both an absolute maximum and an absolute minimum on that interval. In this drill, you investigate what can happen when \( f \) is not continuous or the interval is not closed.

The graph of a function \( f \) is shown at right. Fill in the table below.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Local Maxima in that interval</th>
<th>Local Minima in that interval</th>
<th>Absolute Max in that interval</th>
<th>Absolute Min in that interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>([6,15])</td>
<td>( y = 9 ) (at ( x = 8 ))</td>
<td>( y = 6 ) (occurs at ( x = 12 ))</td>
<td>( y = 10 ) (occurs at ( x = 13 ))</td>
<td>( y = 6 ) (occurs at ( x = 12 ))</td>
</tr>
<tr>
<td>((6,15))</td>
<td>( y = 9 ) (occurs at ( y = 8 ))</td>
<td>( y = 6 ) (occurs at ( x = 12 ))</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>((8,15))</td>
<td>None</td>
<td>( y = 6 ) (occurs at ( x = 12 ))</td>
<td>None</td>
<td>( y = 6 ) (occurs at ( x = 12 ))</td>
</tr>
<tr>
<td>([2,12])</td>
<td>( y = 9 ) (occurs at ( x = 8 ))</td>
<td>None</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>((2,12))</td>
<td>( y = 9 )</td>
<td>None</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>((4,\infty))</td>
<td>( y = 9 )</td>
<td>( y = 6 )</td>
<td>None</td>
<td>None</td>
</tr>
</tbody>
</table>

\( \text{y} = 2 \)
Notice if $f$ is an even degree polynomial with positive leading coefficient.

Reference 1 on page 1 of course packet reminds us that graph goes up on both ends. So no absolute max.

Since $f$ is continuous, with no jumps or holes or vertical asymptotes, we know that there will be a lowest point on graph. So there will be an absolute min. 

least y-value will exist.
But where do we find the absolute min?

Theorem 2 (Locating Absolute Extrema) from Friday tells us that the only places abs min can occur are:
- critical numbers
- x-values that are endpoints of the domain.

In our current example, the domain is $D = (-\infty, \infty)$ so we have no endpoints.

So our absolute min will have to occur at an x-value that is a critical number.

From Friday, we know that the critical numbers for $f$ are $x = 0$, $x = -\sqrt{3}$, $x = \sqrt{3}$.
Find corresponding y-values

\[ f(\sqrt{3}) = (\sqrt{3})^4 - 6(\sqrt{3})^2 + 5 \]
\[ = (3)^2 - 6(3) + 5 \]
\[ = 9 - 18 + 5 \]
\[ = -4 \]

\[ f(-\sqrt{3}) = (-\sqrt{3})^4 - 6(-\sqrt{3})^2 + 5 = f(\sqrt{3}) = -4 \]

\[ f(0) = 0^4 - 6(0)^2 + 5 = 5 \]

**Conclusion:**
Absolute min is \( y = -4 \) (it occurs at \( x = -\sqrt{3} \) and \( x = \sqrt{3} \))

No absolute max!

[End of Lecture]