Chapter 4
Section 4.1 1st Derivatives & Graphs

Book section 4.1 is
- too long
- not well-organized reading
- not well-organized exercises

Today: Horizontal Tangents; Increasing & Decreasing Functions
Correspondence

Behavior of $f'$ at a particular $x = c$ $\iff$ Behavior of the graph of $f$ at $x = c$.

- $f'(c)$ is positive $\iff$ The line tangent to graph of $f$ at $x = c$ tilts upward.

- $f'(c)$ is negative $\iff$ Line tangent to graph of $f$ at $x = c$ tilts downward.

- $f'(c) = 0$ $\iff$ Line tangent to graph of $f$ at $x = c$ is horizontal.
Definition of Increasing on an Interval

"f is increasing on the interval (a, b)."

meaning: If \( a < x_1 < x_2 < b \) then \( f(x_1) < f(x_2) \)

In other words, if you move from left to right in the interval \((a, b)\) the y-values go up.

Similar definition for Decreasing on an Interval

If \( a < x_1 < x_2 < b \) then \( f(x_1) > f(x_2) \).

Examples
New correspondence

Behavior of $f'$ on interval $(a,b)$

$f'$ is positive on interval $(a,b) \implies f$ is increasing on $(a,b)$

$f'$ is negative on interval $(a,b) \implies f$ is decreasing on $(a,b)$

$f'$ is zero on whole interval $(a,b) \iff f$ is constant on $(a,b)$

Compare pages 2 + 4 and see reference 6 in Course Packet

Exercise involving Graph of $f$ $\iff$ Graph of $f'$

Class Drill 14
Use the given graph of $y = f(x)$ to answer the following questions:

(A) Find the intervals on which $f'(x) > 0$.  
\((-\infty, -3) \text{ and } (-3, 1) \text{ and } (1, 3)\)

(B) Find the intervals on which $f'(x) < 0$. 
\((-3, 1) \text{ and } (3, \infty)\)

(C) Find the values of $x$ for which $f'(x) = 0$. 
$x = -3, \ x = 1, \ x = 3$

Then sketch a possible graph of $y = f'(x)$. 
(Compare Class Drill 14 to Class Drill 6 which was done in class on Day 9 (Tues Jan 26).)

**Analytical Example**  
Formula for \( f \) \( \Rightarrow \) info about increasing/decreasing behavior of \( f \)

Let \( f(x) = 2x^3 - 3x^2 - 12x + 5 \)

(A) Find \( x \)-coordinates of points where \( f \) has horizontal tangent lines

(B) Find intervals where \( f \) is increasing/decreasing.

**Solution**

(A) Set \( f'(x) = 0 \) and solve for \( x \)

\[
f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x+1)(x-2)
\]

\( f'(x) = 0 \) when \( x = -1 \) or \( x = 2 \)

\( x \)-values with horizontal tangents on graph of \( f \).

**Check:**

\[
(x+1)(x-2) = x^2 + x - 2x - 2 = x^2 - x - 2 \checkmark
\]
Strategy: Study **Sign Behavior** of \( f' \)

Make sign chart for \( f'(x) = 6(x+1)(x-2) \)

Recall the Partition Numbers for \( f' \) are:
- \( x \)-values where \( f'(x) = 0 \)
- \( x \)-values where \( f' \) is undefined

These are the only places where \( f' \) can change sign.

But since \( f'(x) \) is polynomial, there are no bad \( x \)-values where \( f'(x) \) is undefined.

So the only partition numbers are the ones where \( f'(x) = 0 \) that is \( x = -1, x = 2 \).
Sign Chart for $f'(x) = 6(x+1)(x-2) = 6x^2 - 6x - 12$

Factored Form  Standard Form

\[ f'(x) \quad f'(0) \quad f'(x) \quad f'(2) \quad f'(x) \]

\[ \begin{align*}
F'_{\text{pos}} & \quad f' = 0 & \quad F'_{\text{neg}} & \quad f' = 0 & \quad F'_{\text{pos}} \\
X = -1 & \quad X = 0 & \quad X = 2 & \quad X = 3
\end{align*} \]

Sample
\[ x = -2 \]

Sample
\[ x = 3 \]

\[ f'(-2) = 6(-2+1)(-2-2) = 6(-1)(-4) = \text{pos} \]

\[ f'(0) = 0 + 0 - 12 = \text{neg} \]

\[ f'(3) = 6(3+1)(3-2) = 6(4)(1) = \text{pos} \]

Conclude: $f$ is increasing on intervals $(-\infty, -1)$ and $(2, \infty)$ because $f'$ is positive there.

$f$ is decreasing on the interval $(-1, 2)$ because $f'$ is negative there.

End of lecture