MATH 1350 Day 26 (Wed Feb 24, 2016)

- Pick Up Graded Work
- Sign In

- Math Tutoring Center, in the Math Library on 4th floor of Morton Hall
  - Staffed by Graduate Students in Math who have experience teaching Calculus
  - Hours Tues/Thurs 9am-Noon and 1pm-4pm

- Our Exam 2 is this coming Friday, February 26

Work in groups on Class Drill 12b

\[ C'(t) = \frac{d}{dt} \frac{14t}{t^2 + 1} = (0.14) \left( \frac{d}{dt} \frac{t}{t^2 + 1} \right) = (0.14) \left[ \frac{e^{14t} \left( (t^2 + 1)^2 - t (2t) (t^2 + 1) \right)}{(t^2 + 1)^2} \right] \]

\[ = (0.14) \left[ \frac{9(t^2 + 1) - t (2t)}{(t^2 + 1)^2} \right] = (0.14) \left[ \frac{t^2 + 1 - 2t^2}{(t^2 + 1)^2} \right] \]

\[ = (0.14) \left( -\frac{t^2 + 1}{(t^2 + 1)^2} \right) \]
Class Drill 12b: Rate of Change Problem (Rational Function with Peak)

A drug is administered by pill. The drug concentration (in milligrams per milliliter) in the bloodstream \( t \) hours after the pill is taken is given by the formula

\[
C(t) = \frac{0.14t}{t^2 + 1} \quad \text{for} \quad 0 \leq t
\]

(A) Find \( C(0.5) \) and \( C(3) \). (Give exact answers in symbols and then approximate answers in decimals. Include units in your answer.)

\[
C(0.5) = \frac{0.14(0.5)}{(0.5)^2 + 1} = \frac{0.07}{1.25} = \frac{0.07}{\frac{5}{4}} = \frac{0.07 \times 4}{5} = \frac{0.28}{5} = 0.056
\]

\[
C(3) = \frac{0.14(3)}{3^2 + 1} = \frac{0.42}{10} = 0.042 \quad \text{Exact}
\]

(B) Find \( C'(t) \).

\[
C'(t) = \frac{(0.14)(-t^2+1)}{(t^2+1)^2}
\]

(C) Find \( C'(0.5) \) and \( C'(3) \). (Give exact answers in symbols and then approximate answers in decimals. Include units in your answer.)

\[
C'(0.5) = \frac{0.14(0.5^2+1)}{(0.5^2+1)^2} = \frac{0.14(1.25)}{(1.25)^2} = \frac{0.175}{1.5625} = 0.0172 \quad \text{milligrams per ml}
\]

\[
C'(3) = \frac{(0.14)(-3^2+1)}{(3^2+1)^2} = \frac{0.14(-8)}{10^2} = -0.0112 \quad \text{mg/ml per hour}
\]

(D) Interpret the results of (A) & (C). (Refer to textbook example 6 on page 230 with similar question.)

At time \( t = 0.5 \) hours, concentration is roughly \( 0.056 \) mg/ml and is increasing at a rate roughly \( 0.0172 \) mg/ml per hour.

At time \( t = 3 \) hours, concentration is roughly \( 0.042 \) mg/ml and is decreasing at a rate roughly \( -0.0112 \) mg/ml per hour.

(E) A graph of the concentration is shown below. Illustrate each of the quantities found in questions (A) and (C).
Class Drill 12c: Rate of Change Problem (Rational Function with Horizontal Asymptote)

Bob wrote an i-Phone Calculus app. The sales of the app are modeled by the function

\[ S(t) = \frac{240t^2}{t^2 + 36} \]

In this function, \( t \) is a variable representing time in months since the app was introduced. \( S(t) \) is the total number of apps (in thousands) that have been sold at time \( t \).

(A) Find \( S(6) \). (exact answer)

\[ S(6) = \frac{240(6)^2}{6^2 + 36} = \frac{240 \cdot 36}{36 + 36} = \frac{240 \cdot 36}{72} = \frac{240}{2} = 120 \]

(B) Find \( S'(6) \). (exact answer)

\[ S'(6) = 20 \] See pages (4) and (5)

(C) Interpret the results of (A) & (B). (Refer to textbook example 6 on page 230 with similar question.)

6 months after the app was introduced, a total of 120,000 apps have been sold, and the apps are selling at a rate roughly 20,000 apps per month.

(D) Use the results of (A) and (B) to estimate the total sales after 7 months. (exact answer)

Estimate \( S(7) \) by graph estimate

\[ S(7) \approx 140 \] 140,000 apps

(E) Find the actual value of the total sales after 7 months. (exact answer then approximate answer)

\[ S(7) = \frac{240(7)^2}{7^2 + 36} = \frac{240(49)}{49 + 36} = 138 \] 138,000 apps

(F) How many apps can Bob hope to eventually sell? (exact answer)

\[ \lim_{t \to \infty} S(t) = \lim_{t \to \infty} \frac{240t^2}{t^2 + 36} = \lim_{t \to \infty} \frac{240}{1} = 240 \] 240,000 apps

(G) Illustrate the answers to (A), (B), (D), (E), (F) using the graph below.
Details for Class Drill 12c

\[ S(t) = \frac{240t^2}{t^2 + 36} \]

\[ S'(t) = \frac{d}{dt}\left(\frac{240t^2}{t^2 + 36}\right) = \frac{d}{dt}\left(\frac{240t^2}{t^2 + 36}\right) - \frac{d}{dt}\left(\frac{0}{t^2 + 36}\right) \]

\[ = \frac{(480t)(t^2 + 36) - 240t^2(2t)}{(t^2 + 36)^2} \]

\[ = \frac{480t^3 + (480t)(36) - 480t^3}{(t^2 + 36)^2} \]

\[ = \frac{480(36)t}{(t^2 + 36)^2} \]
\[
5'(6) = \frac{480(36)(6)}{(36^2 + 36)^2} = \frac{480(36)6}{(36+36)^2} = \frac{480(36)(6)}{(2\cdot36)^2}
\]

\[
= \frac{480(36)(6)}{2^2 \cdot 36^2} = \frac{120}{4 \cdot 36} = \frac{120}{6} = 20
\]

*Cancel before multiplying*