Another Example from Section 3.3

\[ f(x) = \frac{x^4 + 4}{x^4} \quad \text{Similar to 3.3 #73} \]

Find \( f'(x) \) by (A) Using the Quotient Rule
(B) First Simplifying \( f \)

Solution

\[ (A) \quad f'(x) = \frac{(d}{dx} (x^4 + 4))x^4 \quad -(x^4 + 4)(\frac{d}{dx} x^4) \]

\[ = \frac{(x^4)^2}{(x^4)^2} \]

\[ = \frac{(4x^3) \cdot x^4}{(x^4)^2} \quad -(x^4 + 4)(4x^3) \]

\[ = 4x^7 - 4x^7 - 16x^3 \]

\[ = \frac{-16x^3}{x^8} \]

\[ = -\frac{16}{x^5} = f'(x) \]

This seems to be a very difficult step.
(B) Rewrite \( f(x) = \frac{x^4 + y}{x^4} = \frac{x^4}{x^4} + \frac{y}{x^4} = 1 + 4x^{-4} \)

Derivative \( f'(x) = \frac{d}{dx} 1 + 4dX^{-4} = 0 + 4(-4x^{-4-1}) \)

\[ = -16x^{-5} = \frac{-16}{X^5} = f'(x) \]

Conclusion: Rewriting \( f \) in a more useful form first, before finding the derivative is

- Very helpful
- Apparently okay to do.
Another example similar to 3.3 #8

\[ f(x) = \frac{3x^5 - 2x^7}{\sqrt[5]{x^3}} \]  

Find \( f'(x) \)

Observe! This could be done with quotient rule, but it would be really messy and prone to errors.

Smarter solution: Rewrite \( f(x) \) in more useful form first

\[ f(x) = \frac{3x^5 - 2x^7}{x^{3/5}} = 3x^{5-3/5} - 2x^{7-3/5} = 3x^{5-3/5} - 2x^{7-3/5} \]

Rewrite

\[ f(x) = 3x^{5/3} - 2x^{7/5} \]

\[ f'(x) = 3 \frac{d}{dx} x^{5/3} - 2 \frac{d}{dx} x^{7/5} = 3 \left( \frac{22}{5} \right) x^{22/5 - 1} - 2 \left( \frac{32}{5} \right) x^{32/5 - 1} \]

\[ = \frac{66}{5} x^{17/5} - \frac{64}{5} x^{27/5} = f'(x) \]
**Tangent Line Problem** Similar to 3.3#65

\[ f(x) = \frac{2x}{3^x} \] Find equation of line tangent to graph of \( f \) at \( x = 3 \).

**Solution** We need to build \( y - f(a) = f'(a)(x-a) \)

**Get Parts**

\[ a = 3 \]

\[ f(a) = f(3) = \frac{2 \cdot 3}{3^3} = \frac{2}{3^2} = \frac{2}{9} = f(a) \]

\[ f'(a) = \frac{(d^2x)3^x - 2x(d3^x)}{(3^x)^2} = \frac{(2)3^x - 2x(3^x \ln(3))}{(3^x)^2} \]

\[ = \frac{2 \cdot 3^x(1-x \ln(3))}{(3^x)^2} = \frac{2(1-x \ln(3))}{3^x} \]

\[ f'(a) = \frac{2(1-3 \ln(3))}{3^3} = \frac{2(1-3 \ln(3))}{27} = f'(a) \]
Assemble Parts

\[ (y - \frac{2}{9}) = \frac{2(1-3\ln(3))}{27} (x - 3) \]

Convert to slope intercept form

\[ y - \frac{2}{9} = \left(\frac{2(1-3\ln(3))}{27}\right)(x) - \left(\frac{2(1-3\ln(3))}{27}\right)3 \]

\[ y = \frac{2(1-3\ln(3))}{27}(x) - \frac{2(1-3\ln(3))}{9} + \frac{2}{9} \]

\[ = \left(\frac{2(1-3\ln(3))}{27}\right)x - \frac{2}{9} + \frac{2\cdot3\ln(3)}{9} + \frac{2}{9} \]

\[ y = \left(\frac{2(1-3\ln(3))}{27}\right)x + \frac{2\ln(3)}{3} \]

End of Lecture