Quiz 3 Today
Exam 1 A week from today, on Fri Feb 5

One Last Example from Section 2.4

Let \( f(x) = \sqrt{x} \). Find \( f'(x) \) using the definition of the derivative.

Solution

\[
\frac{d}{dx} \sqrt{x} = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}
\]

Notice: we cannot substitute \( h = 0 \) at this point because it would lead to \( \frac{0}{0} \).
\[
\begin{align*}
\text{trick!} \quad & \lim_{h \to 0} \quad \frac{\text{"Rationalizing"}}{
\frac{\lim}{h \to 0} \quad \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}
\end{align*}
\]

\[
= \lim_{h \to 0} \frac{\sqrt{x+h} + \sqrt{x} - \sqrt{x} \sqrt{x+h}}{h (\sqrt{x+h} + \sqrt{x})}
\]

\[
= \lim_{h \to 0} \frac{(x+h) - x}{h (\sqrt{x+h} + \sqrt{x})}
\]

\[
= \lim_{h \to 0} \frac{h}{h (\sqrt{x+h} + \sqrt{x})}
\]

Still cannot substitute \( h = 0 \) because it leads to \( \frac{0}{0} \)

But \( h \to 0 \), so \( h \neq 0 \), so we can cancel \( \frac{h}{h} \)
\[
\lim_{{h \to 0}} \frac{1}{{\sqrt{x + h} + \sqrt{x}}}
\]
\[
= \frac{1}{{\sqrt{x} + \sqrt{x}}}
\]
\[
= \frac{1}{{2\sqrt{x}}}
\]

Conclusion: if \( f(x) = \sqrt{x} \) then \( f'(x) = \frac{1}{{2\sqrt{x}}} \)
Harder Example involving $\sqrt{x}$

Let $f(x) = 5 - 17\sqrt{x}$

Find $f'(x)$ using definition of derivative

\[
\frac{f'(x)}{h} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

\[
= \lim_{h \to 0} \frac{(5-17\sqrt{x+h}) - (5-17\sqrt{x})}{h}
\]

\[
= \lim_{h \to 0} \frac{-17\sqrt{x+h} - (-17\sqrt{x})}{h}
\]

\[
= \lim_{h \to 0} (-17) \left( \frac{\sqrt{x+h} - \sqrt{x}}{h} \right)
\]

Factored out $(-17)$

Limit theorem 2.5
Same steps as previous example except that in each step, there is a multiplicative factor \((-17)\) in front.

\[
\frac{-17}{2\sqrt{x}}
\]

Conclusion: if \(f(x) = \frac{5}{2} - 17\sqrt{x}\) then \(f'(x) = \frac{-17}{2\sqrt{x}}\)
Summary:

We have computed $f'(x)$ using the definition of the derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

for these three kinds of functions:

- $f(x)$ polynomial
- $f(x)$ involving $\frac{1}{x}$ hard need common denominator messy but no trick
- $f(x)$ involving $\sqrt{x}$ hard messy need the trick "rationalizing"

These three types are the types that this course covers see suggested exercises. (Do them!)
Section 2.5 Basic Differentiation Properties

Shortcuts to finding \( f'(x) \)

You can use these shortcuts when you are not told to use the definition of the derivative.

The Constant function Rule

If \( f(x) = \text{constant function} \) then \( f'(x) = 0 \)

Example

Let \( f(x) = 5 \) find \( f'(x) \).

Solution \( f'(x) = 0 \) because \( f(x) \) is constant.
Why Does This Make Sense?

Consider graphs

\[ f(x) = 5 \]

\[ f'(x) \]

- On \( f(x) \), all tangent lines have slope \( m = 0 \)
- On \( f'(x) \), all points have \( y \)-value \( y = 0 \)

End of Lecture