Wed Dec 2, 2015

Some notes from Section 11.2 Area

Exercise 10(c) from ch. 11
Find area of trapezoid

Solution:
we have to use a triangulation

\[ \text{red triangle area} = \frac{1}{2} b_1 \cdot h \]
\[ \text{green triangle area} = \frac{1}{2} b_2 \cdot h \]

Sum = trapezoid area = \( \frac{1}{2} b_1 h + \frac{1}{2} b_2 h = \frac{(b_1+b_2)h}{2} \)
Now start Section 11.4 Area of Similar Polygons

Important background:  

**Definition of Similarity**

Suppose \( \triangle ABC \sim \triangle A'B'C' \)

that means \( \frac{\text{side}}{\text{side'}} = \text{same for all pairs} \)

\[
\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'}
\]

(and it also means that all angles are congruent)

But in fact, other important lengths have this same ratio:  

**Theorem 129**

\[
\frac{\text{side}}{\text{side'}} = \frac{\text{altitude}}{\text{altitude'}} = \frac{\text{median}}{\text{median'}} = \frac{\text{angle bisector}}{\text{angle bisector'}}
\]
Here is a bit of a proof of that theorem.

Suppose \( \triangle ABC \sim \triangle A'B'C' \)
and suppose \( D \) and \( D' \) are the feet of the altitudes from vertices \( A \) and \( A' \).

Goal: show that \( \frac{AD}{A'D'} = \frac{\text{side}}{\text{side'}} \)

Notice these triangles

Red angles at \( C, C' \) because original triangles are similar
The green \( \angle \) angles are both right angles,
So \( \triangle A'D'C' \) by AA similarity

So then we can say that the sides have equal ratios \( \frac{AD}{AD'} = \frac{AC}{AC'} \) by definition of similarity.

But these parts \( AD, \ A'D' \) were altitudes of original triangles.

So we have shown that for original triangle \( \frac{\text{altitude}}{\text{altitude'}} = \frac{\text{Side}}{\text{Side'}} \).

End of first part proof.
We use theorem 128 to prove:

**Theorem 135**

If \( \triangle ABC \sim \triangle A'B'C' \)

and \( \frac{\text{side}}{\text{side'}} = r \)

then \( \frac{\text{area}}{\text{area'}} = \left( \frac{\text{side}}{\text{side'}} \right)^2 = r^2 \)
Example

$\overrightarrow{DE} \parallel \overrightarrow{BC}$

$AB = x$
$AD = y$

Given: $\text{Area}(\triangle ABC) = 3(\text{Area}(\triangle DEA))B$

Find $AD$ in terms of $x$.
Find $y$ in terms of $x$.

Solution

Since $\overrightarrow{DE} \parallel \overrightarrow{BC}$

we can say that $\angle ADE \equiv \angle ABC$ by Th.101 applied to parallel lines $\overrightarrow{BC} || \overrightarrow{DE}$

and transversal line $\overrightarrow{AB}$

So $\triangle ABC \sim \triangle ADE$

by AA similarity
By theorem 135
\[ \frac{\text{Area}}{\text{Area'}} = \left( \frac{\text{side}}{\text{side'}} \right)^2 \]

\[ \frac{\text{Area}(ABC)}{\text{Area}(ADE)} = \left( \frac{AB}{AD} \right)^2 = \left( \frac{x}{y} \right)^2 \]

But we are given that the ratio is 3

\[ 3 = \frac{\text{Area}(ABC)}{\text{Area}(ADE)} = \left( \frac{x}{y} \right)^2 \]

\[ \sqrt{3} = \frac{x}{y} \]

\[ y = \frac{x}{\sqrt{3}} \]
Group Work  Book exercise  11.6 #17

Isosceles triangle

$AB = AC = x$
$BD = BC = 1$

Let $y =$ the ratio of areas  
$y = \frac{\text{Area} (\triangle ABC)}{\text{Area} (\odot BCD)}$

Find $y$ in terms of $x$.
Show all details.

End of lecture