Fri Nov 6, 2015

**Quiz 8 will be Mon Nov 9 at start of class.**

**one of the exercises 9.8 [47],[51],[67]**

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**Starting Chapter 9 Euclidean Geometry I: Triangles**

Notice Euclidean geometry Axioms

\[
\begin{align*}
\langle N1 \rangle & : \text{Neutral geom Axioms} \\
\langle N10 \rangle & : \text{Euclidean geom Axioms} \\
\langle EPA \rangle & : \text{Euclidean Parallel Axiom}
\end{align*}
\]

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Since the Neutral geometry Axioms are all on the list of Euclidean geometry Axioms, any theorem that we proved using the Neutral geom Axioms will still be a valid theorem in EuclideanGeom.
So Th. 1 through Th. 96 are valid theorems in Euclidean geom.

Revisit the recurring questions in geometry (p. 30)

Recurring Question #1: Do parallel lines exist?
Recurring Question #2: Given line \( L \) and point \( P \) not on \( L \), how many lines exist that pass through \( P \) and are parallel to \( L \)?

Answer these in Neutral Geometry.

Start with Recurring Question #2.

Given line \( L \) and point \( P \) not on \( L \).

\[ \begin{array}{c}
P \\
\end{array} \]
Consider this sequence of pictures

Given

Line $M$ exists that passes through $P$ and is perp. to $L$.

Line $N$ exists that passes through $P$ and is perp. to $M$.

Th. 74 is the Alternate Interior Angle Theorem of Neutral geometry.

We have used Neutral plane theorems to prove that there exists a line $N$ that passes through $P$ and is parallel to $L$. (at least one line)

Answer to recurring question #2 in Neutral geometry; there is at least one line.
Now what about Reconsider Question #1 in Neutral Geo.

1. Do parallel lines exist?

That is, without being given anything, can we prove that parallel lines exist?

Yes! Consider this.

1. Two distinct points \( A, B \) exist. (by neutral axiom \( N1 \)) \( A \neq B \)
2. There exists a line that passes through \( A, B \). We can call it line \( L \). \( A \in L \) \( B \in L \) (by axiom \( N2 \))
3. There exists a point that is not on line \( L \). We can call the point \( P \). (by axiom \( N3 \))
(5) Line $M$ exists (by Th. 60)

(5) Line $N$ exists (by Th. 86)

(6) $L \parallel N$ (by Th. 74)

End of proof
So in Neutral Geometry, the answers to the recurring questions are:

**Question #1** Yes, parallel lines do exist.

**Question #2** At least one line.

That is, if we let the variable \( n \) be the number of parallel lines that pass through \( P \) and are parallel to \( L \), then in Neutral Geometry, \( n \geq 1 \), at least one line.
Now switch to considering **Euclidean Geometry**

The Euclidean Parallel Axiom \(<EPA>\) tells us that \(n \leq 1\).

(Not more than one parallel)

Combine the facts that:

* \(n \geq 1\) (by Neutral Axiom Facts)
* \(n \leq 1\) (by \(<EPA>\))

And we get the \(P\) theorem

**Theorem 97** In Euclidean Geometry, \(n = 1\).
Easy Corollaries

Th. 98 (corollary)

In Euclidean geometry,
If a line intersects one of two parallel lines,
then it also intersects the other.

That is,
If $L, M$ are parallel and line $T$ intersects $M$,
than $T$ also intersects $L$. 
Discuss Alternate Interior Angle Theorem and Its Converse

Th 7.74
The Alt Int Angle theorem (of Neutral geom)
Given lines L, M cut by transversal T.
If all interior angles are congruent then the lines are parallel.

Th 7.74 Neutral Geometry
The converse is not a theorem in Neutral Geometry. But it can be proven in Euclidean.

Theorem 100
Converse of Alternate Interior Angle Theorem
Given lines $L, M$ cut by transversal $T$

If $L \parallel M$

Then alternate interior angles are congruent

Euclidean geometry