Mon Nov 2, 2015

**Finishing Chapter 8 Circles**

8.4 Stuff about 10th angle bisectors

Th. 91 about points on the bisector of an angle

Given \( \triangle ABC \)

\( P \) is in bisector of \( \angle BAC \)

\( D \) is equidistant from sides \( AB, AC \)
Justify the steps in the proof of the following theorem:

Theorem 91 about points on the bisector of an angle in Neutral Geometry
Given: Neutral Geometry, angle $\angle BAC$, and point $D$ in the interior of the angle
Claim: The following statements are equivalent
(i) $D$ lies on the bisector of angle $\angle BAC$.
(ii) $D$ is equidistant from the sides of angle $\angle BAC$.

Proof
(1) In Neutral Geometry, suppose that point $D$ lies in the interior of angle $\angle BAC$. (Make a drawing.)

Proof that (i) $\Rightarrow$ (ii)
(2) Suppose that (i) is true. That is, suppose that $D$ lies on the bisector of angle $\angle BAC$. (Make a new drawing.)

(3) Let point $E$ be the foot of the perpendicular from $D$ to line $\overrightarrow{AB}$, and let point $F$ be the foot of the perpendicular from $D$ to line $\overrightarrow{AC}$. (Make a new drawing.)

(4) Then $\triangle DAE \cong \triangle DAF$. (Justify.) (Make a new drawing.)

(5) So $DE \cong DF$. (Justify.) (Make a new drawing.)

(6) Conclude that $D$ is equidistant from the sides of angle $\angle BAC$. That is, (ii) is true.

Proof that (ii) $\Rightarrow$ (i)
(7) Suppose that (ii) is true. That is, suppose that \( D \) is equidistant from the sides of angle \( \angle BAC \). (Make a new drawing.)

(8) Let point \( E \) be the foot of the perpendicular from \( D \) to line \( \overline{AB} \), and let point \( F \) be the foot of the perpendicular from \( D \) to line \( \overline{AC} \). (Make a new drawing.)

(9) Then \( \overline{DE} \cong \overline{DF} \). (Justify.) (Make a new drawing.)

(10) Then \( \triangle DAE \cong \triangle DAF \). (Justify.) (Make a new drawing.)

(11) So \( \angle DAE \cong \angle DAF \). (Justify.) (Make a new drawing.)

(12) Conclude that \( D \) lies on the bisector of angle \( \angle BAC \). That is, (i) is true.

End of Proof
Concurrence theorems

Theorems that state that some collection of lines is concurrent.

We will study a few of these in the course.

Our first one is Th. 92: In neutral geom

- Three angle bisectors are concurrent at a point that we can call G.
- G is equidistant from the three triangle sides.
Class Drill 11: Drill for Section 8.4 Concurrence of Angle Bisectors of a Triangle

Theorem 92 In Neutral Geometry, the three angle bisectors of any triangle are concurrent at a point that is equidistant from the three sides of the triangle.

Proof

(1) In Neutral Geometry, suppose that triangle ΔABC is given. (Make a drawing.)

Show that the bisectors of ∠A and ∠B intersect.

(2) There exists a ray $\overline{AD}$ that bisects ∠CAB. (Justify.) (Make a new drawing.)

(3) Point D lies in the interior of angle ∠CAB. (Justify.) (Make a new drawing.)

(4) Ray $\overline{AD}$ intersects side $\overline{BC}$ at a point that we can call E. (Justify.) (Make a new drawing.)

(5) There exists a ray $\overline{BF}$ that bisects ∠ABE. (Justify.) (Make a new drawing.)

(6) Point F lies in the interior of angle ∠ABE. (Justify.) (Make a new drawing.)

(7) Ray $\overline{BF}$ intersects segment $\overline{AE}$ at a point that we can call G. (Justify.) (Make a new drawing.) We have shown that the bisectors of ∠A and ∠B intersect at G.
Consider distances from the point of intersection to the sides of the triangle

(8) The distance from $G$ to line $\overline{AC}$ equals the distance from $G$ to line $\overline{AB}$. (Justify.) (Make a new drawing.)

By Th. 91 (i) $\rightarrow$ (ii)

applied to

point $G$

on bisector of $\angle BAC$

(9) The distance from $G$ to line $\overline{BA}$ equals the distance from $G$ to line $\overline{BC}$. (Justify.) (Make a new drawing.)

By Th 91 (i) $\rightarrow$ (ii)

applied to

point $G$

on bisector of $\angle ABC$

(10) So the distance from $G$ to line $\overline{CA}$ equals the distance from $G$ to line $\overline{CB}$. (Justify.) (Make a new drawing.)

(by (8), (9), transitivity)

(11) Therefore, point $G$ lies on the bisector of $\angle BCA$. (Justify.) (Make a new drawing.)

Th. 91 (ii) $\rightarrow$ (i)

applied to point $G$

which is known to be equidistant from $\overline{CA} + \overline{CB}$

(12) We have shown that the bisectors of all three angles $\angle A$ and $\angle B$ and $\angle C$ intersect at $G$ and that $G$ is equidistant from the three sides of the triangle.

End of Proof
Section 8.5  Tangent lines & inscribed circles

Th. 93  In Neutral geometry

![Diagram showing tangent lines to circles]

**Proof**

![Diagram with labeled given and proved statements]

Th. 78 (ii) → (i)

The tangent lines are perpendicular to the radial segment.
definition of circle

\[ \overline{PB} \cong \overline{PC} \]

by HL theorem 7.1

Applied to
Hypotenuses \( \overline{PA} \cong \overline{PA} \)
Legs \( \overline{PB} \cong \overline{PC} \)

\[ \triangle PBA \cong \triangle PCA \]

definition of what it means to say the triangles are congruent

Tangent segments
\[ \overline{AB} \cong \overline{AC} \]

End of proof
Final theorem of chapter

Th. 95 In Neutral Geometry
Every triangle has exactly one inscribed circle

Proof

Point $G$ exists that is equidistant from the sides

There is a circle centered at $G$ that passes through those three points

End of Lecture