Fri Oct 30, 2015  Continuing Ch 8 Circles

Section 8.3 Theorems About Chords

Most of the theorems are proven easily, using theorems about triangles.

Recurring trick: In any situation involving circles, whether or not radial segments are mentioned, the radial segments exist and they are all the same length. So isosceles triangles are lurking everywhere.

Example: Given

\[ \text{chord} \quad \text{chord} \quad \text{isosceles triangle} \]

The isosceles triangles can often be used in proofs.
Example of using this trick

Prove Theorem 8.7  Any perpendicular from the center of a circle to a chord bisects the chord.

What is often the most tricky part of proving a theorem is separating the given stuff from the claim. If the theorem is presented as an "IF-Then" statement (a "conditional" statement) then that job is easy. But many theorems are not presented in conditional form. In class we discussed strategies for recognizing how to translate theorems like the one presented above into the form of a conditional statement. For the theorem above, a conditional form could be

Theorem 8.7  If a line passes through the center of a circle and is perpendicular to a chord then the line bisects the chord.
In pictures, we would write

\[ \text{Th. 87} \]

It is productive to write a proof in pictures.

**Proof**

1. **Given**
   - \( \overline{AB} \) and \( \overline{AC} \) exist

2. **Segments**
   - \( \overline{AB}, \overline{AC} \) exist

3. **Definition of circle**
   - \( \overline{AB} \cong \overline{AC} \)

4. **Theorem 85** ii \( \Rightarrow \) iii about isosceles triangles

5. **Conclusion**
   - \( \overline{AD} \) bisects \( \overline{BC} \)
Similar Theorem 88
The segment joining the center of a circle to the midpoint of a chord is perpendicular to the chord.

Translate into a conditional statement:
If a segment joins the center of a circle to the midpoint of a chord then the segment is perpendicular to the chord.

Th. 88

you provide the steps & justifications & illustrations for the proof. As with proof of Th. 87, this proof will involve an isosceles triangle.
Th. 89 The perpendicular bisector of a chord passes through the center of the circle.

Rephrase as a conditional statement:

If a line \( L \) is the perpendicular bisector of a chord, then \( L \) passes through the center of the circle.
Proof

Given

Line L is the perpendicular bisector of chord AB.

Center of circle exists by def of circle.

A line m exists that passes through A and is perpendicular to BC.

Observe that the line M that we conjured up with Th. 66:
- Goes through center of circle
- Is perpendicular to BC
- Bisects BC.
But since $M$ is perp. to $BC$

a Bisects $\overline{BC}$

That tells us that line $M$ must actually be the same line as our old line $L$.

So our old line $L$ actually goes through the center of the circle, too.

- Exam 3 moved to Wed Nov 3

- Optional Take Home Bonus Quiz Due Mon Nov 1
  1. (5 points) Justify + illustrate steps in proof of Th. 91
  2. (5 points) Justify + illustrate steps in proof of Th. 92

(you can print out blanks from the course web page. They are in the calendar for today as Class Drills)