Quiz 7 will be at end of class today.

Optional Bonus Take-Home Quiz Due Wednesday at the start of class:

5pts Illustrate drawing the steps in proof of Theorem 8.5 (Chapter 8)
5pts Prove Theorem 8.6 (Chapter 8)

Exam 3 will be a Week from Today, Mon Nov 2

Section 7.7 Parallel Lines in Neutral Geom

None of the ten Neutral Geometry Axioms say anything about parallel lines.
No mention of whether or not parallel lines ever exist.

Two chapters from now, in Ch. 9, we will introduce the "Euclidean Parallel Axiom" (EPA). It will be our 11th axiom.

Eleven Axioms of "Euclidean Geometry"

N1

N10

EPA

We certainly will now some stuff about parallel lines then, because we will have that axiom EPA.
But even now, in Neutral Geometry

\[ N_1 \subseteq N_10 \]

Neutral Geom Axioms

It turns out that we can prove that parallel lines exist, and we can say things about them. (without using EDA)

This is very important historically.

Was not known for sure until mid-1800s.

We will discuss new terminology: transversals, interior angles, alternate interior angles, corresponding angles. Definition 6.1
Theorem 73 Eight Equivalent Statements about two lines & transversal in \( \mathbb{E} \), Neutral plane.

Theorem does not say that any of the statements are true. Only that they are all true or are all false.

Either this

\[ \text{T} \]

-or this

\[ \text{T} \]

all 8 statements are true

all 8 statements are false.
Example of how to prove some of the equivalences:

Steps:
1. \( \angle ABE \cong \angle BFE \)

\[ \triangle \]

\[ \triangle \]

4. \( \angle ABH \cong \angle DEH \)

Prove that (1) \( \rightarrow \) (4):

1. Assume (1) is true so \( \angle ABE \cong \angle BFE \)

2. We know \( \angle FEB \cong \angle DEH \)
   By vertical angles from (3)

3. Therefore \( \angle ABE \cong \angle DEH \) by transitivity

So statement (4) is true

End of proof
Diagram of proof structure

(Viii) (i) (ii) (iii)
(Vii)
(Vi)
(V)
(iv)

Challenge. For Exam 3, you will be asked to prove one of the arrows on this diagram.
Theorem 74: Alternate Interior Angle Theorem for Neutral Geometry
Theorem 74 The Alternate Interior Angle Theorem for Neutral Geometry

**Given:** Neutral Geometry, lines $L$ and $M$ and a transversal $T$

**Claim:** If a pair of alternate interior angles is congruent, then lines $L$ and $M$ are parallel.

**Contrapositive:** If $L$ and $M$ are not parallel, then a pair of alternate interior angles are not congruent.

**Proof (Indirect proof by method of contraposition)**

(1) Suppose that in Neutral Geometry, lines $L$ and $M$ and a transversal $T$ are given, and that $L$ and $M$ are not parallel. *(make a drawing)*

(2) Let $A$ be the point of intersection of lines $L$ and $M$, let $B$ be the point of intersection of lines $L$ and $T$, and let $C$ be the point of intersection of lines $M$ and $T$. *(update drawing)*

(3) There exists a point $D$ such that $A * B * D$. *(Make a new drawing)*

(4) Observe that $\angle CBD$ is an exterior angle for $\triangle ABC$, and $\angle BCA$ is one of its remote interior angles. *(Make a new drawing)*

(5) $m(\angle CBD) > m(\angle BCA)$. *(Justify.) (Make a new drawing)*

(6) Observe $\angle CBD$ and $\angle BCA$ are alternate interior angles and they are not congruent. That is, lines $L, M, T$ do not have the special angle property.

**End of Proof**