Quiz 7 will be in-class on Monday (Oct 26). You will be asked to illustrate and justify the steps in one of these theorems. (I will choose which theorem)

- Theorem 64: The Triangle Inequality
- Theorem 66: About perpendicular lines
- Theorem 71: The Hypotenuse-Leg Congruence Theorem

Continuing Section 7.3: Bigger vs. Smaller Parts of Triangles

So far we have proven:  
\[ CS \rightarrow CA \text{ Th. 52 } \]  
\[ CA \rightarrow CS \text{ Th. 55 } \]  
\[ BS \rightarrow BA \text{ Th. 61 } \]
Next: Theorem 62 \( BA \rightarrow BS \)

I will only discuss the proof structure.

Proof is interesting for three reasons:

- It gets by just recycling previous results. No new geometric drawings.
- It proves the contrapositive \( \neg BS \rightarrow \neg BA \).
- It uses proof by division into cases.

Read the proof in the book.

Combine the two theorems:

\[ \text{Th.61 } BS \rightarrow BA \]
\[ \text{Th.62 } BA \rightarrow BS \]

In any triangle, bigger angles are always opposite bigger sides.
Final Theorem from Section on Bigger & Smaller Parts in Triangles

**Theorem 64: The Triangle Inequality**

In Neutral geometry, the length of any side of any triangle is less than the sum of the lengths of the other two sides.

That is, for all non-collinear points $A$, $B$, $C$, the inequality $AC < AB + BC$ is true.

Study the proof in the book. This is one of the three candidates for a quiz question for Monday's Quiz.
Section 7.4 Advanced Topics. We will skip this chapter. (But we get to use the theorems.)

Section 7.5 Perpendicular lines.

There is a very fundamental pair of theorems about perpendicular lines.

Old Theorem 46 Existence and uniqueness of perpendicular lines in the case where some point $P$ lies on line $L$.

New Theorem 66 Existence and uniqueness of perpendicular lines in the case where point $P$ does not lie on $L$. This line exists and is unique.
Study the book proof work on the justification.
It could be one of the questions on Monday's quiz.
The proof is a very important style involving "mirror copies" of triangles.

Two easy Theorems

Theorem 67: The shortest segment from a point to a line is the perpendicular segment.

Proof: Exterior angle theorem gives us BABA tells us 90 Small  Big

Perpendicular Segment  non-perpendicular segment
This is also the key to proving Th. 68. In any right triangle, the hypotenuse is the longest side.

Now that we have theorem 66:

Given

We can introduce Triangle Altitudes

\[ \text{Altitude Segment} \]

\[ \text{Altitude Segment} \]

\[ \text{Altitude Segment} \]

\[ \text{Altitude line} \]
Section 7.6 A Final Look at Triangle Congruence in Neutral Geom

Th. 70 AAS Congruence

Proof is in book. We won't study it ("Advanced" Proof.)

Th. 71 HL Congruence (Hypotenuse-Leg Congruence)

This is the 3rd possibility of a problem for Monday's Quiz.
End of Lecture