Ch. 7  The Axiom of Triangle Congruence.

Four behaviors of triangles in drawings

If these parts match

SAS

ASA

SSS

AAS

then the triangles can be superimposed
If we want all this stuff to happen in our abstract geometry, we might expect to have to specify all this behavior in axioms.

But in fact, we only have to specify one of these behaviors in an axiom.

**Axiom (N10) SAS Axiom**

The remaining three behaviors can be proven to happen in theorems.

The ASA Congruence Theorem (Th. 54)
The SSS Congruence Theorem (Th. 58)
The AAS Congruence Theorem (Th. 70)
We haven't really talked about congruence much in class.

Congruent line segments have the same length.

Congruent angles have the same measure.

Congruent triangles

“Corresponding” sides are congruent.

“Corresponding” angles are congruent.

In other words “corresponding” parts are congruent.

But we need to be clear about what we mean by “corresponding parts”.
Axiom \( \angle N10 \) can be illustrated this way:

\[ \triangle \rightarrow \angle N10 \rightarrow \triangle \]

Section 7.2 Theorems about Correspondences in Triangles

Theorem 52 CS \( \Rightarrow \) CA Theorem

(The Isosceles Triangle Theorem)

In Neutral Geometry

If two sides of a triangle are congruent, then the two angles opposite those sides are also congruent.

That is, if CS then CA.
Proof: Transpose the copy.

Given isosceles triangle \( \triangle ABC \) with \( AB \cong AC \).

Notice that \( \triangle ABC \) and \( \triangle A'BC' \) are two symbols that refer to the same triangle.

But we can treat them as if they are two different triangles: Left triangle \( \triangle ABC \)
Right triangle \( \triangle A'CB' \)
introduce a correspondence of vertices

vertices of left triangle vertices of right triangle

\((A, B, C) \leftrightarrow (A, C, B)\)

then we have these automatic correspondences of parts

\[\overrightarrow{AB} \leftrightarrow \overrightarrow{AC}\]

\[\angle BAC \leftrightarrow \angle CAB\]

\[\angle AC \leftrightarrow \overline{AB}\]

\[\overline{AB} \cong \overline{AC} \text{ (given)}\]

\[\angle BAC \cong \angle CAB \text{ (reflexive property of angle congruence. Every angle is congruent to itself)}\]

\[\overline{AC} \cong \overline{AB} \text{ (given) + itself}\]
By Axiom (110) (SAS) applied to $\triangle ABC$ and $\triangle ACB$, we can say that therefore, all other corresponding parts are also congruent, that is the two triangles are congruent.

In particular, $\angle ABC \cong \angle ACB$

Left triangle

Right triangle

This tells us that in our original triangle, $\angle ABC \cong \angle ACB$

End of Proof
Justifying steps in proof of Th. 54: The ASA congruence theorem

Class Drill

Old Theorem that we never covered in class

Th. 24 Congruent Segment Construction

\[ \overrightarrow{AB} \]

Here exists point \( E \)

Point \( E \) exists that make a segment \( AB \)

Justification for Th. 54 Proof Step 2

Applied Th. 24 Congruent Segment Construction theorem

applied to given segment \( \overrightarrow{ED} \)

given ray \( \overrightarrow{EB} \)

Theorem gives us a point on ray \( \overrightarrow{EB} \) We can call that point \( G \).
Justify the steps in the proof of the following theorem. (Your justifications may refer to any prior theorem and to Axioms <N1> through <N10>.) See the proof in the textbook for drawings.

Theorem 54 the ASA Congruence Theorem for Neutral Geometry
In Neutral Geometry, if there is a one-to-one correspondence between the vertices of two triangles, and two angles and the included side of one triangle are congruent to the corresponding parts of the other triangle, then all the remaining corresponding parts are congruent as well, so the correspondence is a congruence and the triangles are congruent.

Remark: The statement of the theorem can be illustrated by the picture below.

Proof
(1) Suppose that \( \triangle ABC \) and \( \triangle DEF \) are given such that \( \angle ABC \cong \angle DEF \) and \( BC \cong EF \) and \( \angle BCA \cong \angle EFD \).

(2) There exists a point \( G \) on ray \( ED \) such that \( EG \cong BA \). (Justify.)

\( \text{Theorem 24 Congruent Segment Construction} \)
\( \text{applied to} \)
\( \text{given segment } BA \)
\( \text{given ray } ED \)
\( \text{tells us that a point } G \text{ exists on ray } ED \)
\( \text{such that } EG \equiv BA \)
(We suspect that \( G \) is the same point as \( D \), but we have not yet proven that, so we should not draw it that way.)

(3) \( \triangle GEF \cong \triangle ABC \). (Justify.)

(4) \( \angle EFG \cong \angle BCA \). (Justify.)
(5) \( \angle EFG \cong \angle EFD \). (Justify.)

(6) Points \( D \) and \( G \) are on the same side of line \( EF \). (Justify.)

(7) Ray \( FD \) must be the same ray as \( FG \). (Justify.)

(8) Line \( DF \) can only intersect line \( DE \) at a single point. (Justify.)

(9) Points \( D, G \) must be the same point. (Justify.)

(10) \( \triangle DEF \cong \triangle ABC \). (Justify.)

End of proof
Optional Bonus Take-Home Quiz Due Monday October 19: Finish this class Dr. 11
That is, justify steps (3) - (10) in the proof of Theorem 54.
This will be due Monday at the start of class.
It will be worth 8 points.