Mon Sep 28, 2015

Leftover ideas about line segments & betweenness

Definition 25. Line segment $\overline{AB}$

idea: using a "good" coordinate function for $\overrightarrow{AB}$
that is, a coordinate function $f$ such
that $f(A) = 0$ and $f(B)$ is positive.

the segment $\overline{AB}$ is defined as the set

$$\overline{AB} = \{ P : P \in \overrightarrow{AB} \text{ such that } 0 \leq f(P) = f(B) \}$$

that is, $P \in \overline{AB}$ such that
either $P = A$ (so $f(P) = 0$)
or $A \neq P \neq B$ (so $0 < f(P) < f(B)$)
or $P = B$ (so $f(P) = B$)
Observe that if \( P \) is on segment \( \overline{AB} \) and \( P \) is known to \( \not\) be \( A \) or \( B \), then \( A \neq P \neq B \) is true.

Put this into a conditional statement

Suppose we are given that \( P \neq A \) and \( P \neq B \), then the following are equivalent:
(i) \( P \) is on segment \( \overline{AB} \)
(ii) \( A \neq P \neq B \).

Either both are true or both are false

Now discuss the statements of separation axiom \( \langle N_6 \rangle \)
Discuss Class Dr. 11

Proof of theorem 27

We are ready to justify statement (ii)

(a) Point D is in halfplane $H_c$ ($D$ & $C$ are on same side of line)

To prove this we will need to prove that $D$ passes the test has the property mentioned in the definition of $H_c$.

To prove this statement we have four possible tools:

Axiom $\langle N6\rangle^{(ii)}$

$\langle N6\rangle^{(ii)}$ contrapositive

$\langle N6\rangle^{(iii)}$

$\langle N6\rangle^{(iv)}$ contrapositive

Only this statement ends by saying two points are in same halfplane.

We need to use this axiom.
(N6) (iii) contrapositive says:

If \( \overline{PQ} \) does not intersect line \( L \),
then points \( P, Q \) are on same side of line \( L \).

Adapt that statement to our current situation:
we want to show that \( D, C \) are on same side of the line \( L \).

The tool that we can use is this version of
\(<N6> (iii) contrapositive:

If \( \overline{CD} \) does not intersect \( L \)
then \( C, D \) are on same side of \( L \).
So our strategy to prove \(C \neq D\) are on some side of line \(L\) should be the following:

- First somehow prove that \(CD\) does not intersect \(L\).
- Then use axiom (N6)(iii) contrapositive to say that therefore, \(C, D\) are on same side of \(L\).
  
  (In the same half-plane.)
Introduce point $D$.
(9) There exists a point such that $A * C * Point$. (Justify.)

(10) This point cannot be the same as any of our previous three points. (Justify.)

So it must be a new point. Call it $D$. So $A * C * D$.
(Make a new drawing.)

(11) Point $D$ is in half-plane $H_C$. (Justify.)

**Part 1**

Prove that $CD$ does not intersect line $L_1 = \overline{AB}$

Observe that $A$ is on line $CD$ because $A * C * D$.

We also know that $A$ is on line $L_1$ from earlier.

So line $CD$ and line $L_1$ intersect at $A$.

**Theorem 1** says that two distinct lines that intersect can only intersect at one point.

So line $CD$ and line $L_1$ only intersect at $A$.

And we know that $A$ is not on segment $CD$ because $A * C * D$.

Conclude that line $L_1$ and segment $CD$ do not intersect.

**Part 2**

By axioms (N6)(iii) contrapositive, $CD$ are on same side of line $L_1$. 
Now look ahead in class drill 4.

Step (14) is similar to step 11.

Goal in step 14:
Prove that $E$ is in half plane $H$.
That proves that $E,C$ are on same side of line $l$.
Strategy will be same as for step 11.

- Prove that segment $EC$ does not intersect line $l$.
- Then use Axiom (No)(iii) contrapositive to say that therefore, $E,C$ are on same side of $l$. 
Introduce point $E$.

(12) There exists a point such that $B \ast C \ast \text{Point}$.

(13) This must be a new point.

Call it $E$. So $B \ast C \ast E$.

(Make a new drawing.)

(14) Point $E$ is in half-plane $H_c$.

Strategy: *Prove that segment $EC$ does not intersect line $L$.
* Then use axiom $(N6)(iii)$ continuing to say that therefore, $E, C$ are on same side of $L$.

Conclusion of Part I:

(15) Points $C$ and $D$ and $E$ are non-collinear.

Remark: Notice that in steps (4), (11), (14) we saw that points $C, D, E$ are in half-plane $H_c$. 

Take-home Quiz (5 points) Due this coming Wed Sep 30

Illustrate & Justify the steps in the proof of Theorem 28 (Pasch’s Theorem)

End of Lecture