**Friday, Aug 28, 2015**

**Finishing Section 1.2**

More concise illustration of the independence of the 5-ake axiom system

<table>
<thead>
<tr>
<th>Interpretations</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Truth of Axioms</td>
<td></td>
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</tr>
<tr>
<td>\langle 1 \rangle</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>\langle 2 \rangle</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>\langle 3 \rangle</td>
<td>T</td>
<td>T</td>
<td>T</td>
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</tr>
</tbody>
</table>

**Interpretations #1, 2 show that axiom \langle 1 \rangle can be either true or false while \langle 2 \rangle, \langle 3 \rangle are both true. That is, Axiom \langle 1 \rangle is independent.**

Similar explanations for independence of axioms \langle 2 \rangle, \langle 3 \rangle conclude: the set of axioms is independent.
Leftover topic from Section 1.1: Isomorphic Models.

Recall Bob's model for 5-ake axiom system

\[ \text{akes} \leftrightarrow \text{dots} \]
\[ \text{bems} \leftrightarrow \text{segments} \]
\[ \text{ake is related to the bem} \leftrightarrow \text{dot touches the segment} \]

Here's another model: Carol's Model

\[ \text{akes} \leftrightarrow \text{letters } a, b, c, d, e \]
\[ \text{bems} \leftrightarrow \text{sets } \{a, b, c\}, \{a, c\}, \{a, d\}, \{a, e\}, \{b, c\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\} \]
\[ \text{ake is related to the bem} \leftrightarrow \text{the letter is in the set} \]
These models are "kind of the same". That is, they are isomorphic.

Two models are called isomorphic if there is a one-to-one correspondence between primitive terms and relations of one model and the primitive terms and relations of the other model. Such that all relationships among objects are preserved.

By example: for the 5-ake axiom system, Bob's model is isomorphic to Carol's model.
Section 1.3 Property of Axiom Systems: Completeness

Define an axiom system is **complete** if all models for that axiom system are **isomorphic**.

An axiom system is called **incomplete** if there exist models for it that are not isomorphic.

Example 5: the axiom system is complete.

But proving completeness can be very difficult.
Example of an incomplete axiom system

Primitive terms: ake, ben

Primitive relations: the ake is related to the ben.

Axioms:
1. There are exactly 5 akes
2. For each two distinct akes there exists exactly 1 ben that both akes are related to.

Examples of models

Model #1
- 5 ake dots
- 10 ake-segments

Model #2
- 5 ake dots
- 1 ben-segment

Model #3
- 5 dots
- 5 segments
The 3 models are not isomorphic.

(They can't be, because they don't even have the same number of segments.)

Alternate way of describing incompleteness of an axiom system: consider whether or not there are any independent statements that can be said about the axiom system.

Example For the axiom system on page 5, consider statement #1: There exist exactly 10 beams.

Notice that this is an independent statement, because it is true in model #1, but false in models #2, #3.
The fact that there exists an independent statement that can be said about the page 5 axcin system tells us that the axcin system is not complete.

Another example for page 5 axcin system, consider statement #2 there is exactly one bcn.

Again, note that this is an independent statement.

(It is true in model #2, but false in models #1, #3).

Another example for page 5 axcin system, consider statement #3 there are exactly 5 bcn.

This is independent.

Class: Can you think of any other independent statements about the page 5 axcin system?