Class Drill 9: Drill for Section 7.7: The Alternate Interior Angle Theorem

Theorem 74 The Alternate Interior Angle Theorem for Neutral Geometry

**Given:** Neutral Geometry, lines $L$ and $M$ and a transversal $T$

**Claim:** If a pair of alternate interior angles is congruent, then lines $L$ and $M$ are parallel.

**Contrapositive:** If $L$ and $M$ are not parallel, then a pair of alternate interior angles are not congruent.

**Proof (Indirect proof by method of contraposition)**

1. Suppose that in Neutral Geometry, lines $L$ and $M$ and a transversal $T$ are given, and that $L$ and $M$ are not parallel. *(make a drawing)*

2. Let $A$ be the point of intersection of lines $L$ and $M$, let $B$ be the point of intersection of lines $L$ and $T$, and let $C$ be the point of intersection of lines $M$ and $T$. *(update drawing)*

3. There exists a point $D$ such that $A \ast B \ast D$. *(Make a new drawing)*

4. Observe that $\angle CBD$ is an exterior angle for $\triangle ABC$, and $\angle BCA$ is one of its remote interior angles. *(Make a new drawing)*

5. $m(\angle CBD) > m(\angle BCA)$. *(Justify.)* *(Make a new drawing)*

6. Observe $\angle CBD$ and $\angle BCA$ are alternate interior angles and they are not congruent. That is, lines $L, M, T$ do not have the special angle property.

**End of Proof**