Class Drill 4: Drill for Section 5.1 Justifying and illustrating Steps in proof of Theorem 27

Theorem 27: Given any line, each of its half-planes contains at least three non-collinear points.

Proof

(1) Given any line, call it $L_1$. (Make a drawing.)

Introduce points $A$ and $B$.

(2) There exist two distinct points on $L_1$. (Justify.)

Call them $A$ and $B$. (Make a new drawing.)

Part I: Introduce Half-Plane $H_C$ and show that it contains three non-collinear points.

(3) There exists a point not on $L_1$. (Justify.)

Call it $C$.

(Make a new drawing.)

(4) Point $C$ lies in one of the two half-planes determined by line $L_1$. (Justify.)

Call it $H_C$.

(Make a new drawing.)
**Introduce line** $L_2$.

(5) There exists a unique line passing through $A$ and $C$. (Justify.)

(6) The line passing through $A$ and $C$ is not $L_1$. (Justify.)

So it must be new. Call it $L_2$.

(Make a new drawing.)

**Introduce line** $L_3$.

(7) There exists a unique line passing through $B$ and $C$. (Justify.)

(8) The line passing through $B$ and $C$ is not $L_1$ or $L_2$. (Justify.)

So it must be new. Call it $L_3$.

(Make a new drawing.)
Introduce point $D$.
(9) There exists a point such that $A \ast C \ast \text{Point}$. (Justify.)

(10) This point cannot be the same as any of our previous three points. (Justify.)

So it must be a new point. Call it $D$. So $A \ast C \ast D$.
(Make a new drawing.)

(11) Point $D$ is in half-plane $H_C$. (Justify.)
Introduce point $E$. 
(12) There exists a point such that $B \ast C \ast Point$. (Justify.)

(13) This must be a new point. (Justify.)

Call it $E$. So $B \ast C \ast E$.
(Make a new drawing.)

(14) Point $E$ is in half-plane $H_C$. (Justify.)

Conclusion of Part I:
(15) Points $C$ and $D$ and $E$ are non-collinear. (Justify.)

Remark: Notice that in steps (4), (11), (14) we saw that points $C, D, E$ are in half-plane $H_C$. 