Fri Dec 4, 2015  (Last Day!!)
Continuing Section 6.2

Producers' Surplus.

On Wednesday, we saw that the price demand curve is decreasing for consumers.

\[ P = D(x) \] demand curve.

\( (\text{small } x, \text{ large } p) \)

\( (\text{large } x, \text{ small } p) \)

\[ P = \text{selling price} \]
\[ x = \text{number of items purchased (the demand)} \]
for suppliers there is a different relationship between $p = \text{price}$ and $x = \text{number of items made (the supply)}$

for suppliers,
when price $p$ is small, the number of items made, $x$, will be small, because there is not much incentive to produce the item.

But if the price $p$ is large, the number of items made, $x$, will be large, because there is a large incentive to produce the item.

Relationship between $p$, price, $x$, supply

The price supply curve $p = S(x)$

Graph will be increasing.
Consider the situation where a particular selling price $\bar{P}$ has been established. All the producers who would have been willing to sell for a lower price think they are getting a great deal.

The green area represents all the money that producers get above what they would have been willing to sell for. This is called the Producers' Surplus.

Integral formula for Producers' Surplus

$$PS = \int_{x=0}^{x=x} (\bar{P} - S(x)) \, dx$$
Example of Finding Producers’ Surplus

7-2#48
Find the Producers’ Surplus at a price level of $\bar{p} = 55$ for the price supply function $p = S(x) = 15 + 1.1x + .003x^2$

Solution:

\[ PS = \int_{x=0}^{x=\bar{x}} (\bar{p} - S(x))\,dx = \int_{x=0}^{x=55} (55 - (15 + 1.1x + .003x^2))\,dx \]

Need to figure out $\bar{x}$

\[ p = 15 + 1.1x + .003x^2 \]

\[ \bar{p} = 15 + 1.1\bar{x} + .003(\bar{x})^2 \]

55 = 15 + 1.1\bar{x} + .003(\bar{x})^2

How can we figure out $\bar{x}$??

Subtract 55 from both sides

\[ .003\bar{x}^2 + 1.1\bar{x} - 40 = 0 \]

Use quadratic formula

\[ \bar{x} = \frac{-1.1 \pm \sqrt{(1.1)^2 - 4(.003)(-40)}}{2(.003)} \]
\[ \bar{x} = -1 \pm \sqrt{0.01 + 0.48} \]
\[ = -1 \pm \sqrt{0.49} \]
\[ = -1 \pm 0.7 \]

Can't have a negative \( \bar{x} \), so must use \( +0.7 \).

\[ \bar{x} = -1 + 0.7 \]
\[ = \frac{0.6}{0.006} \]

\[ \bar{x} = 100 \]
\[ P_S = \int_{x=0}^{x=100} 55 - (15 + 0.1x + 0.003x^2) \, dx \]

\[ = \int_{x=0}^{x=100} 40 - 0.1x - 0.003x^2 \, dx \]

\[ = \left. \left( 40x - \frac{0.1x^2}{2} - \frac{0.003x^3}{3} \right) \right|_{x=0}^{x=100} \]

\[ = \left( 40(100) - 0.05(100)^2 - 0.001(100)^3 \right) - \left( 0 \right) \]

\[ = 4000 - 0.05(10000) - 0.001(1000000) \]

\[ = 4000 - 500 - 1000 \]

\[ = 2500 \]
Equilibrium Price + Equilibrium Quantity

The point $(\bar{x}, \bar{p})$ where the $D(x)$ and $S(x)$ curves cross is called the equilibrium point.

It represents the point where the number of items consumers are willing to buy equals the number of items that producers are willing to produce.
Examples of finding Equilibrium Point

Let $D(x) = 25 - .004x^2$

$S(x) = 5 + .004x^2$

Find Equilibrium Point

Solution

Set $S(x) = D(x)$ and solve for $x$

$25 - .004x^2 = 5 + .004x^2$

$20 = .008x^2$

$x^2 = \frac{20}{.008}$

$x = \pm \sqrt{\frac{20}{.008}} = \pm \sqrt{2500}$

$x = 50$
Suppose that
- the price demand equation is \( p = D(x) = 9 - x^2 \)
- the price-supply equation is \( p = S(x) = 3 + x \).

Draw the graphs of the two equations on the grid below. Be sure to label the graphs with their corresponding equations.

Label the coordinates of the equilibrium point using a label of the form \((\bar{x}, \bar{p}) = (\ , \ )\).

Shade the region corresponding to the consumers' surplus and label it \( CS \).

Shade the region corresponding to the producers' surplus (using a different shade) and label it \( PS \).
Find the consumers' surplus. (Simplify the integrand before integrating!)

\[ CS = \int_{x=0}^{x=x^*} [D(x) - \bar{p}] \, dx = \int_{x=0}^{x=2} (9 - x^2) - 5 \, dx \]

\[ = \int_{x=0}^{x=2} 4 - x^2 \, dx \]

\[ = \left(4x - \frac{x^3}{3}\right) \bigg|_0^2 \]

\[ = \left(4(2) - \frac{(2)^3}{3}\right) - (0) \]

\[ = 8 - \frac{8}{3} = \frac{16}{3} = CS \]

Find the producers' surplus. (Simplify the integrand before integrating!)

\[ PS = \int_{x=0}^{x=x^*} [\bar{p} - S(x)] \, dx = \text{Use geometry} = \frac{1}{2} (2)(2) = 2 = PS \]

*end of lecture*