Chapter 5
Section 5.1 Antiderivatives

Definition of Antiderivative
- Words: F is an antiderivative of f.
- Meaning: f is the derivative of F.
  That is \( F' = f \)
- Arrow diagram

\[ \begin{align*}
  F & \quad \text{take derivative} \\
  \rightarrow & \quad f
\end{align*} \]
Example #1 \[ F(x) = \frac{x^3}{3} \] is an antiderivative of \( f(x) = x^2 \)

Check: \[ F'(x) = \frac{d}{dx} \left( \frac{x^3}{3} \right) = \left( \frac{1}{3} \right) \frac{d}{dx} x^3 = \left( \frac{1}{3} \right) \cdot 3x^2 = x^2 = f(x) \]

Example #2 Is \( G(x) = \frac{x^3}{3} + 17 \) an antiderivative of \( f(x) = x^2 \)?

Explain.

Solution

Check \[ G'(x) = \frac{d}{dx} \left( \frac{x^3}{3} + 17 \right) = \frac{d}{dx} \frac{x^3}{3} + \frac{d}{dx} 17 = \frac{1}{3} \cdot 3x^2 + 0 = x^2 = f(x) \]

Conclusion: \( G \) is an antiderivative of \( f \).
Observation: There is more than one antiderivative of \( f(x) \).
That is, given some function \( F(x) \) that is known to be an antiderivative of \( f(x) \), we can make lots of other antiderivatives.
Any function of the form \( F(x) + C \) where \( C \) is a constant will also be an antiderivative of \( f(x) \).

Theorem 1: These are the only antiderivatives of \( f(x) \).
That is, if one antiderivative of \( f(x) \) is \( F(x) \), then all antiderivatives are of the form \( F(x) + C \).
Graph some of the antiderivatives of $f(x) = x^2$.

Notice the family of antiderivatives are all vertical translations of one another.
Consider arrow diagram for antiderivative

\[ F \xleftarrow{\text{take derivative}} G \xrightarrow{\text{find all antiderivatives}} F \]

\[ F(x) + C \]
More Examples

Example #3 (like 5.1 #29)

Is \( F(x) = \frac{(5x+7)^3}{3} \) an antiderivative of \( f(x) = (5x+7)^2 \)?

**Solution:** 

\[
F'(x) = \frac{d}{dx} \left( \frac{(5x+7)^3}{3} \right) = \frac{1}{3} \frac{d}{dx} (5x+7)^3 = \frac{1}{3} \cdot 3(5x+7)^2 \cdot 5 = (5x+7)^2 \cdot 5 \]

So \( F(x) \) is *not* an antiderivative of \( f(x) \).
Example 4 (similar to 5.1/#27)

Is \( F(x) = x \ln(x) - x + 5 \) an antiderivative of \( f(x) = \ln(x) \)?

Solution

Find \( F(x) \)

\[
F'(x) = \frac{d}{dx}(x \ln(x) - x + 5)
\]

\[
= \frac{d}{dx}(x \ln(x)) - \frac{d}{dx}x + \frac{d}{dx}5
\]

\[
= \frac{d}{dx}(x \ln(x)) - 1 + 0
\]

\[
= \frac{d}{dx}x \ln(x) + x \frac{d}{dx} \ln(x)
\]

\[
= \ln(x) + \frac{1}{x} - 1
\]

\[
= \ln(x) + 0 - 1
\]

\[
= \ln(x)
\]

\[
= F(x)
\]

Conclude \( F(x) \) is an antiderivative of \( f \).
Indefinite Integrals

Definition

- Words: the indefinite integral of \( f(x) \)
- Symbol: \( \int f(x) \, dx \)
- Meaning: the symbol represents the family of all antiderivatives of \( f(x) \).

Remark: Since we know that given one antiderivative \( F(x) \), we can represent the whole family by writing \( F(x) + c \) where \( c \) is a constant that can be any real number.

That is if \( F'(x) = f(x) \) then \( \int f(x) \, dx = F(x) + c \)
New arrow diagram

\[ F(x) + C \quad \text{derivative} \quad f(x) \quad \text{indefinite integral} \]
Indefinite Integrals of Basic Functions

The Power Rule for indefinite integrals

if \( n \neq -1 \), then \( \int x^n \, dx = \frac{x^{n+1}}{n+1} + C \)

Example Find \( \int x^8 \, dx \)

Solution

\[ \int x^8 \, dx = \frac{x^{8+1}}{8+1} + C = \frac{x^9}{9} + C \]

Check by differentiation,

\[ \frac{d}{dx} \left( \frac{x^9}{9} + C \right) = \left( \frac{1}{9} \right) x^9 + 0 = x^8 \checkmark \]

End of Lecture