Mon Oct 26, 2015

Section 4.6 Optimization

Optimization Problems are simply Max/Min Problems.

Possible Complications

- May be presented as word problems
- Function is generally not given. You may need to figure out what function to use.
- Domain is generally not specified. You may have to figure it out.
- Once you figure out the domain, you may find that it is not a closed interval.
- May involve more than one variable.

There is not one approach that works for all problems.
Today: Examples involving Perimeter & Area.

Example #1 Similar to Suggested Exercises 4.6 #9, 17

Find positive numbers \(x, y\) such that
- the sum \(2x + y = 900\)
- the product is maximized.

Solution

**Step 1** Identify Equation I: \(2x + y = 900\)

**Step 2** Identify Equation II: \(x \cdot y = P\) the product.

Our goal is to find \(x, y\) that maximize \(P\).

**Step 3** Eliminate one of the variables

Solve Equation I for \(y\) in terms of \(x\)

\[y = 900 - 2x\] new Equation I
Substitute the New Equation I into Equation II

\[ xy = p \]
\[ x(900 - 2x) = p \]

Substituted in
\[ y = 900 - 2x \]
from equation 1.

Step 4 Observations

This equation involves \( P + x \), and it is solved for \( P \) in terms of \( x \). So it describes \( P \) as a function of the variable \( x \).

\[ P = x(900 - 2x) \]

Equation from
\[ P(x) = x(900 - 2x) \]

Function from
\[ = x \cdot 2(450 - x) \]
\[ = 2x(450 - x) \]
\[ = 900x - 2x^2 \]

Factored form

Standard form
Step 5: Consider (or determine) the domain.

We know that \( x > 0 \) so domain is definitely restricted to \( x > 0 \).

But \( y \) must be \( y > 0 \) also:

\[
\text{must have } y = 900 - 2x
\]

must be greater than 0.

When \( x = 450 \) we would have \( y = 900 - 2(450) \)

\[
= 900 - 900 = 0
\]

if \( x > 450 \) then \( y \) would go negative.

So domain is \( 0 < x < 450 \).
Step 6 Use calculus to maximize \( P(x) = 900x - 2x^2 \)
on the interval \( 0 < x < 450 \).

**Solution**

Notice: \( P \) is continuous but interval is not closed. We are not guaranteed a max or a min.

But if there is a max or min, it would have to happen at a critical number.

So, find critical numbers for \( P(x) \).

Start by finding partition numbers for \( P(x) \).

\[
P(x) = 900x - 2x^2
\]

\[
P'(x) = 900 - 2(2x) = 900 - 4x
\]
Notice \( P'(x) \) is polynomial so there are no \( x \)-values that cause \( P(x) \) to not exist.

So look for \( x \)-values that cause \( P'(x) = 0 \)

\[
0 = P'(x) = 900 - 4x
\]

\[
4x = 900
\]

\[
\boxed{x = 225}
\]

\( x = 225 \) is a critical number for \( P' \) because \( P(225) = 0 \).

Is \( x = 225 \) also a critical number for \( P \)?
Must check does \( P(225) \) exist?
Yes because \( P(x) \) is polynomial

So \( x = 225 \) is the only critical number for \( P(x) \).
Since \( P(x) \) is downward-facing parabola, we know that \( x = 225 \) is the location of the absolute max.

Step 6 Find corresponding value of \( y \) and the product \( P \).

We know \( y = 900 - 2x \) from equation I

\[
= 900 - 2(225) \\
= 900 - 450 \\
= 450
\]

The resulting product is \( P = xy = 225 \cdot 450 \)

\[
= 101,250
\]
Example #2 Similar to Sugg exercises 4.6 #34, 35

Fence Problem

Farmer needs to build a fence to make a rectangular corral next to a barn. (He only needs to fence 3 sides.)

He has 900 feet of fencing.

What dimensions will make the largest possible area?

Solution
Equation expressing the amount of available fencing

\[ 2x + y = 900 \quad \text{Equation \#1} \]

Equation expressing the area

\[ A = xy \quad \text{Equation \#2} \]

So we need to find positive numbers \( x, y \) such that

a. \( 2x + y = 900 \quad \text{Equation \#1} \)

b. \( A = xy \) is maximized \( \text{Equation \#2} \)

Using result of previous example, we know \( x = 225, y = 450, A = 101,250 \) is the best fence to build.