Section 4.5 Absolute Extrema

Definition of Absolute Max

Words: The absolute max of function $f$.
Words: The absolute max of $f$ on domain $D$.

Meaning: The absolute max is

- a $y$-value $y$
- it occurs at at least one $x$-value in the domain $D$. So $y = f(c)$ for at least one $x = c$ in domain $D$,
- It is the greatest $y$-value anywhere on the domain. That is if $x \in D$ then $f(x) \leq f(c) = y$

$X$ is in the Domain
Where can absolute max occur?

Theorem 2 Locating Absolute Extrema (page 294)

The only x-values where absolute max or min can occur are x-values that are

- endpoints of the domain (if the domain has endpoints)

or x-values that are critical numbers that are in the domain

This theorem does not guarantee that absolute max and mins occur. And in general, for a function f, there might not be a max or min.
There is one situation where it is guaranteed that there will be an absolute max and absolute min.

Theorem 1: The Extreme Value Theorem

If a function $f$ satisfies these two requirements
- the domain is a closed interval $[a, b]$
- the function $f$ is known to be continuous on $[a, b]$

Then you are guaranteed that
$f$ will have an absolute max on the interval
$f$ will have an absolute min on the interval
Suppose a function $f$ has a domain that is a closed interval, and $f$ is continuous on the interval. So, Theorem 1 guarantees that $f$ will have an absolute max and absolute min. Where would you look for them?

By Theorem 2, the absolute max and min can only occur at:
- endpoints of the interval
- critical numbers in the interval.
This brings us to

The Closed Interval Method for finding absolute extrema of a continuous function on a closed interval.

- Confirm that the domain is a closed interval $[a, b]$.
- Confirm that $f$ is continuous on $[a, b]$ (explain how you know).
- Find the critical numbers for $f$.
- Make a list of important $x$-values
  - Important $x$-values
    - $x = a$ endpoint
    - $C_1$
    - $C_2$
    - $C_k$ critical numbers in the interval
    - $x = b$ endpoint
0. Find the corresponding y-values

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>f(a) = ....</td>
</tr>
<tr>
<td>c</td>
<td>f(c) = ....</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>c_k</td>
<td>f(c_k) = ....</td>
</tr>
<tr>
<td>b</td>
<td>f(b) = ...</td>
</tr>
</tbody>
</table>

0. Identify the greatest y-value and least y-value and state conclusion clearly in a sentence.

"The absolute max is y = Blah. It occurs at x = Blah."
"The absolute min is y = Blah. It occurs at x = Blah."
Example \( f(x) = x^4 - 6x^2 + 5 \)

Find absolute extrema on interval \([-3, 2]\)

Solution

Interval \([-3, 2]\) is closed

\( f \) is continuous because it is a polynomial

Find critical numbers

\[ f(x) \]

Set \( f'(x) = 0 \)

Solve for \( x \).

\[ f'(x) = 4x^3 - 12x \]

\[ = 4(x^3 - 3x) \]

\[ = 4x(x^2 - 3) \]

\[ = 4x(x + \sqrt{3})(x - \sqrt{3}) \]

Partitions numbers for \( f'(x) \) are

\[ x = 0 \quad x = -\sqrt{3} \quad x = \sqrt{3} \]

Check

\[ (x + \sqrt{3})(x - \sqrt{3}) = x^2 + \sqrt{3}x - x\sqrt{3} - 3 \]

\[ = x^2 - 3 \]