Quiz 17 at start of class
Please stay in your desk after you finish.
- put quiz face-down on your desk
- keep your stuff put away (including your phone)
Look at your book

typo: $+4,5,6,7$

should be $+7$
Textbook has lots of examples of polynomial functions and quite a few rational functions. But there is a shortage of other kinds of functions. In particular, a shortage of problems involving exponential functions.

Here is a function type guaranteed to be on a future quiz, exam, or final.

\[ f(x) = x \cdot e^{-x} \]  

This is the type that you will see on a quiz or exam.
(A) find $f'(x)$ and make a sign chart for $f'(x)$
(B) find intervals of increase and decrease
(C) find $x$-coordinates of all local max + mins (say which type, max or min)
(D) find the $y$-coordinates of all local extrema.
(E) find $f''(x)$ and make a sign chart for $f''(x)$
(F) find intervals of concavity
(G) find $x$-coordinates of all inflection points.
(H) find corresponding $y$-coordinates of all inflection points
Solution:

(A) \( f'(x) = \frac{d}{dx} \left( x \cdot e^{(-x)} \right) \)

\[ = \left( \frac{dx}{dx} \right) e^{(-x)} + x \left( \frac{d}{dx} e^{-x} \right) \]

\[ = (1) e^{(-x)} + x (e^{(-x)}) \]

\[ = (1) e^{(-x)} - xe^{(-x)} \]

\[ = (1 - x)e^{(-x)} \]

Product rule

Rewrite

\( e^{(x)} = e^{(x+0)} \)

Exponential Rule #2

\( \frac{d}{dx} e^{(kx)} = ke^{(kx)} \)

cleaned up

observe common factor

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Look for partition numbers for \( f'(x) \):

- Any \( x \)-values that cause \( f'(x) \) to not exist? No
- Any \( x \)-values that cause \( f'(x) = 0 \)?
\[ 0 = (1-x) e^{(-x)} \]

\[ x=1 \text{ will cause this factor to be zero} \]

never zero because \( e^{\text{anything}} > 0 \)

So the only partition number for \( f'(x) \) is \( x=1 \) because it causes \( f'(1) = 0 \)

Sign chart for \( f'(x) = (1-x) e^{(x)} \)

\[ f'(0) = 0(1-0)e^{(0)} = \text{pos} \times \text{pos} = \text{pos} \]

\[ f'(2) = (1-2)e^{(2)} = \text{neg} \times \text{pos} = \text{neg} \]
(B) $f$ is increasing on interval $(-\infty,1)$ because $f'$ is positive.

C) **Local max at $x=1$**

D) Y-coordinate is $f(1) = 1 \cdot e^{(-1)} = 1 \cdot \frac{1}{e} = \frac{1}{e}$

Local max of $y = \frac{1}{e}$ occurs at $x=1$.

E) $f''(x) = \frac{d}{dx} f'(x) = \frac{d}{dx} \left((1-x)e^{-x}\right)$ product rule

$$= \left(\frac{d}{dx}(1-x)\right)e^{-x} + (1-x)\frac{d}{dx}e^{-x}$$

$$= (-1)e^{-x} + (1-x)(-e^{-x})$$

$$= (-1)e^{-x} + (-1)e^{-x} - xe^{-x}$$

$$= xe^{-x} - 2e^{-x}$$

Some calculation as before

Cleaning up common factor!
\[ f''(x) = (x - 2) e^{-x} \]

Look for partition numbers for \( f''(x) \):
- No \( x \)-value that causes \( f''(x) \) to not exist.
- \( x = 2 \) is the only \( x \)-value that causes \( f''(x) = 0 \).

Sign chart for \( f''(x) = (x - 2) e^{-x} \):

\[
\begin{array}{c|c|c|c}
\text{Interval} & f''(x) & f''(x) = 0 & f''(x) \\
\hline
(-\infty, 2) & - & - & + \\
(2, +\infty) & + & + & + \\
\end{array}
\]

Samples:
- \( f''(1) = (1 - 2) e^{-1} = -1 e^{-1} = \text{neg} \)
- \( f''(3) = (3 - 2) e^{-3} = e^{-3} = \text{pos} \)

Conclusion:
- \( f''(x) < 0 \) for \( x < 2 \) and \( f''(x) > 0 \) for \( x > 2 \).
- \( f''(x) = 0 \) at \( x = 2 \).
- \( f''(x) > 0 \) implies that \( f(x) \) is concave up, and \( f''(x) < 0 \) implies that \( f(x) \) is concave down.
(F) \( f \) concave down on interval \((-\infty, 2)\) because \( f'' \) is neg.
\( f \) concave up on interval \((2, \infty)\) because \( f'' \) is pos.

(G) **inflection point at \( x=2 \)**

(H) the y-coordinate is \( f(2) = 2 \cdot e^{(-2)} = 2 \cdot \frac{1}{e^2} = \frac{2}{e^2} \)

Inflection point is \((x, y) = (2, \frac{2}{e^2})\)

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\text{end of class}