Remember: Optional Bonus Take-Home Quiz due in class tomorrow (Wednesday).

- Check your OU e-mail to get the quiz.
- Print it out and work on it.
- Staple it.
- Due at the start of class tomorrow

Continuing Example from yesterday \( f(x) = 3\sqrt{x^2-3x+21} \)

(B) Find equation of line tangent to graph of \( f(x) \) at \( x=4 \)

Solution: We need to build this \( (y-f(a)) = f'(a)(x-a) \)

Get Parts:

\[ a = 4 \]

\[ f(a) = f(4) = 3\sqrt{(4)^2-3(4)+21} = 3\sqrt{16-12+21} = 3\sqrt{4+21} \]

\[ = 3\sqrt{25} = 3(5) = 15 \]

\( x \)-coordinate of point of tangency

\( y \)-coordinate of point of tangency
\[ f'(x) = \frac{3(2x-3)}{2\sqrt{x^2-3x+21}} \quad \text{(from yesterday)} \]

\[ f'(a) = f'(4) = \frac{3(2(4)-3)}{2\sqrt{4^2-3(4)+21}} = \frac{3(8-3)}{2.5} = \frac{3 \times 5}{2.5} = \frac{3 \times 2}{2} \]

Substituting into tangent line equation:

\[ (y - 15) = \frac{3}{2} (x - 4) \]

Point slope form of equation of tangent line.

Convert to slope intercept form:

\[ y - 15 = \frac{3}{2} (x - 4) \]

\[ y = \left(\frac{3}{2}\right)x - \left(\frac{3}{2}\right)4 + 15 \]

\[ y = \left(\frac{3}{2}\right)x - 6 + 15 \]

\[ y = \left(\frac{3}{2}\right)x + 9 \quad \text{tangent line equation} \]
Continuing Section 3.4 Chain Rule

Examples so far have had power function outer functions. Today! Examples where outer function is not power function.

Example where outer function a logarithmic function

Find \( f'(x) \) for \( f(x) = 7 \ln(5x^2 - 30x + 65) \)

Solution:

\[
\frac{d}{dx} \ln(5x^2 - 30x + 65) \quad \text{used Constant Multiple Rule}
\]

\[
= 7 \cdot \frac{d}{dx} \ln(5x^2 - 30x + 65)
\]

\[
= 7 \cdot \frac{d}{dx} (5x^2 - 30x + 65)
\]

\[
= 7 \cdot \frac{d}{dx} \left( 5 \cdot (x^2 - 6x + 13) \right)
\]

\[
= 7 \cdot \frac{d}{dx} (5x^2 - 30x + 65)
= 7 \cdot \frac{d}{dx} (5x^2) + \frac{d}{dx} (-30x) + \frac{d}{dx} (65)
= 7 \cdot (10x) - 30 + 0
= 70x - 30
\]

Outer \( (\cdot) = \ln(\cdot) \) Empty Versions

Outer' \( (\cdot) = \frac{1}{(\cdot)} \)

Chain Rule Details

Inner \( (x) = 5x^2 - 30x + 65 \)

Inner' \( (x) = \frac{d}{dx} (5x^2 - 30x + 65) = 10x - 30 \)

Outer \( (\cdot) = \ln(\cdot) \)

Outer' \( (\cdot) = \frac{1}{(\cdot)} \) Empty Versions

\[
= \frac{7(10x - 30)}{5x^2 - 30x + 65} = \frac{7(2x - 6)}{5(x^2 - 6x + 13)} = \frac{7(2x - 6)}{(x^2 - 6x + 13)} = \frac{14(x - 3)}{(x^2 - 6x + 13)}
\]
Examples where outer function is an exponential function

Recall Exponential Function Derivative Rule #2

Earlier in the course, I just gave you this rule. Instead, derive the rule using the chain rule.

\[
\frac{d}{dx} e^{(kx)} = d\text{outer}(\text{inner}(x)) \quad \text{chain rule}
\]

\[
= \text{outer}'(\text{inner}(x)) \cdot \text{inner}'(x)
\]

\[
= e^{(kx)} \cdot k
\]

\[
= ke^{(kx)}
\]

This agrees with Exponential Function Rule #2.

\[
\frac{de^{(kx)}}{dx} = ke^{(kx)}
\]
Example: \( f(x) = e^{-(x^2 + 4x - 4)} \)

(A) Find equation of line tangent at \( x = 0 \)

Solution: We need to build \((y - f(a)) = f'(a)(x-a)\)

Get parts

\( a = 0 \)

\( f(a) = f(0) = e^{-(0^2 + 4(0) - 4)} = e^{-4} = \frac{1}{e^4} \)

\( f'(x) = \frac{d}{dx} e^{-(x^2 + 4x - 4)} \)

\[ f'(x) = \frac{d}{dx} e^{d \text{outer}(\text{inner}(x))} \]

\[ = \text{outer}'(\text{inner}(x)) \cdot \text{inner}'(x) \]

\[ = e^{-(x^2 + 4x - 4)} \cdot (-2x + 4) \]

\( f'(a) = f'(0) = e^{-(0^2 + 4(0) - 4)} \cdot (-2(0) + 4) = e^{-4} \cdot 4 = \frac{1}{e^4} \cdot 4 = \frac{4}{e^4} \)
Substitute parts into the equation:

\[ y - f(a) = f'(a)(x-a) \]

\[ y - \frac{1}{e^4} = \left( \frac{4}{e^4} \right) (x-0) \]

Convert to slope-intercept form:

\[ y - \frac{1}{e^4} = \left( \frac{4}{e^4} \right) x \]

\[ y = \left( \frac{4}{e^4} \right) x + \frac{1}{e^4} \]
Graph these Results

Hey, this is a "Bell Curve"

(B) Find x-coordinate of all points that have horizontal tangent lines.

Solution set $n = f'(x) = 0$ and solve for $x$

$0 = f'(x) = e^{(-x^2+4x-4)}(-2x+4)$

at least one of these factors must be 0.

recall anything = 0

So it must be that $-2x+4 = 0$

$x = 2$
Remark: "Bell Curves" are functions of the form 
\[ e^{\text{second-degree polynomial with negative leading coefficient}}. \]

For example, in the current problem, we have 
\[ f(x) = e^{-x^2 + 4x - 4} \]

The graphs have this characteristic form:
Bell Curve

[end of lecture]