Mon Sep 28, 2015

Friday we saw \( \frac{d}{dx} \ln(x) = \frac{1}{x} \)

Does this make sense graphically?

Recall graph of \( y = \ln(x) \)

Start with graph of \( y = e^x \)

\( e^0 = 1 \)
\( e^1 = e \)
\( e^{1/2} = \sqrt{e} \)

Interchange all \( x \) and \( y \) to get graph of \( y = \ln(x) \)

\( \ln(1) = 0 \)
\( \ln(e) = 1 \)
\( \ln(\frac{1}{e}) = -1 \)

\( e \)
\( 1 \)
\( (e, 1) \)

Domain: all \( x \geq 0 \)
Range: all \( y \geq 0 \)

Asymptote: \( x = 0 \)

Domain: all \( x \geq 0 \)
Range: all real numbers \( y \)
Now consider graph of \( y = \frac{1}{x} \)

\[ (-2, -\frac{1}{2}) \quad (-1, -1) \quad (-\frac{1}{2}, -2) \]

\[ (\frac{1}{2}, 2) \quad (1, 1) \quad (2, \frac{1}{2}) \quad (3, \frac{1}{3}) \]

Domain: all \( x \neq 0 \)
Range: all \( y \neq 0 \)

Now consider our derivative equation:

\[
\frac{d}{dx} \ln(x) = \frac{1}{x}
\]

\( \ln(x) \) has domain \( x > 0 \)
So its derivative can only have domain \( x > 0 \)

equation valid for \( x > 0 \)
Now consider why the equation \( \frac{d}{dx} \ln(x) = \frac{1}{x} \) for \( x > 0 \) is believable.
Important remark about $\ln(x) = \frac{1}{x}$

It's not true!

The relationship is

$$\frac{d \ln(x)}{dx} = \frac{1}{x}$$
Tangent line example

Find the equation of the line tangent to
\[ f(x) = 5 + \ln(x^3) \] at \( x = e^2 \)

Solution:
Simplify our function using a rule of logarithms:
\[ \ln(x^3) = 3 \ln(x) \]
So \( f(x) = 5 + 3 \ln(x) \)

Find equation of line tangent at \( x = e^2 \)

We need to build this equation:
\[ (y - f(a)) = f'(a)(x-a) \]
Get parts

\[ a = e^2 \]

this is the x-coordinate of the point of tangency.

\[ f(a) = f(e^2) = 5 + 3 \ln(e^2) = 5 + 3(2) \quad \text{because } \ln(e^2) = 2 \]

\[ = 5 + 6 \]

\[ = 11 \]

this is the y-coordinate of the point of tangency.

\[ f'(x) = \frac{df(x)}{dx} = \frac{d}{dx} \left( 5 + 3 \ln(x) \right) \]

\[ = 0 + 3 \frac{d \ln(x)}{dx} \]

\[ = 0 + 3 \left( \frac{1}{x} \right) \]

\[ = \frac{3}{x} \]
\[ f'(a) = f'(e^2) = \frac{3}{e^2} = \text{slope of tangent line} \]

Substitute parts into equation:

\[ (y - f(a)) = f'(a)(x-a) \]

\[ (y - 11) = \frac{3}{e^2} (x - e^2) \]

Convert to slope intercept form:

\[ y - 11 = \frac{3}{e^2} (x - e^2) \]

\[ = \left( \frac{3}{e^2} \right) x - \left( \frac{3}{e^2} \right) e^2 \]

\[ y - 11 = \left( \frac{3}{e^2} \right) x - 3 \]

\[ y = \left( \frac{3}{e^2} \right) x + 8 \]
Section 3.3 The Product Rule

Used for finding the derivative of a product.

\[ f(x) \cdot g(x) \]

The obvious thing:

\[ \frac{d}{dx} (f(x) \cdot g(x)) = \frac{df(x)}{dx} \cdot \frac{dg(x)}{dx} \]

The obvious thing is wrong!

The correct method

The Product Rule

\[ \frac{d}{dx} (f(x) \cdot g(x)) = \left( \frac{df(x)}{dx} \right) g(x) + f(x) \left( \frac{dg(x)}{dx} \right) \]
Examples

Find derivative of \( f(x) = (-3x^2 + 5x - 7)(3x - 2) \)

Using Product rule

Solution: \( f'(x) = \left( \frac{d}{dx}(-3x^2 + 5x - 7) \right)(3x - 2) + \left( -3x^2 + 5x - 7 \right) \frac{d}{dx}(3x - 2) \)

By the Product rule

\[
= (-6x + 5 - 0)(3x - 2) + (-3x^2 + 5x - 7)(3 - 0)
\]

\[
= (-6x + 5)(3x - 2) + (-3x^2 + 5x - 7)3
\]

\[
= -18x^2 + 12x + 15x - 10 - 9x^2 + 15x - 21
\]

\[
= -27x^2 + \frac{42x}{42x} - 31
\]
Example \( f(x) = (-3x^2 + 5x - 7)e^x \) find \( f'(x) \)

Solution

\[
\frac{d}{dx} \left( -3x^2 + 5x - 7 \right) e^x + \left( -3x^2 + 5x - 7 \right) \frac{d}{dx} e^x
\]

\[
= (-6x + 5)e^x + (-3x^2 + 5x - 7)e^x
\]

Common factor of \( e^x \)

\[
= \left[ (-6x + 5) + (-3x^2 + 5x - 7) \right] e^x
\]

\[
= (-3x^2 - x - 2)e^x
\]

Simplified form

\[ \text{end of lecture} \]