Fri Sep 25, 2015

Containing Section 3.2 Derivatives of Exponential and Logarithmic Functions

Last Time we discussed Continuously Compounded Interest

Balance \( A = Pe^{rt} \)

Consider \( P, r \) as constants. Then this gives \( A \) as a function of the variable \( t \).

Balance Function \( A(t) = Pe^{rt} \)

Last time we discussed

Rate of change of the balance \( A'(t) = \frac{dA(t)}{dt} = \cdots = rPe^{rt} \)

Notice that \( A'(t) = r \cdot A(t) \)
Example: Deposit $1000 into an account that earns 2% interest compounded continuously.

(A) What is the balance after 5 years?

Solution: \( A(t) = Pe^{rt} \)

\[
A(5) = 1000e^{0.02(5)}
\]

\approx 1105.17

(B) What is the rate of change of the balance at time \( t = 5 \) years?

Solution: We need to find \( A'(5) \).

We know \( A'(t) = rPe^{rt} \) from earlier.

So, \( A'(5) = 0.02(1000)e^{0.02(5)} \)

\approx 22.0 \text{ dollars per year.} \)
Example

The value of a truck is given by the formula

\[ S(t) = 174,000 \times 0.9^t \]

t is the time in years since the truck was purchased.

(A) What is the purchase price of the truck?

Solution: The truck was purchased at time \( t = 0 \)

Its value then was

\[ S(0) = 174,000 \times 0.9^{0} = 174,000 \]

(B) What is the value after 5 years?

Solution: \( S(5) = 174,000 \times 0.9^{5} \approx \$102,745.26 \)

The exact answer is: \( 174,000 \times 0.9^{5} \)

Approximate answer

(C) What is the rate of change of the value of the truck after 5 years?
Solution: We need \( S'(5) \)

Strategy: Find \( S'(t) \)

Substitute \( t = 5 \).

\[
S'(t) = \frac{d}{dt} 174,000 (0.9)^t
\]

\[
= 174,000 \frac{d}{dt} (0.9)^t
\]

\[
= 174,000 (0.9)^t \ln(0.9)
\]

\[
S'(5) = 174,000 (0.9)^5 (\ln(0.9))
\]

\[
\approx -10825.30
\]

Notice that this is negative!
$S(t)$

$(0, 174,000)$

$M = -10,825$ dollars per year

$(5, 102,745)$
Derivatives of Logarithmic Functions

New derivative rules

Log function Rule #1: \[
\frac{d}{dx} \ln(x) = \frac{1}{x}
\]

Log function Rule #2: \[
\frac{d}{dx} \log_b(x) = \frac{1}{x \ln(b)}
\]

Examples: Find these derivatives

(A) \( f(x) = 12 \ln(x) \)

Solution: \[
f'(x) = 12 \cdot \frac{d}{dx} \ln(x) = 12 \left( \frac{1}{x} \right) = \frac{12}{x}
\]
(b) \( f(x) = 12 \log_{13}(x) \)

Solution: \( f'(x) = 12 \cdot \frac{d}{dx} \log_{13}(x) = 12 \cdot \left( \frac{1}{x \ln(13)} \right) = \frac{12}{\ln(13) \cdot x} \)

(c) \( f(x) = 12 \log(x) \)

Solution: In our book, \( \log(x) \) means \( \log_{10}(x) \)

So, \( f(x) = 12 \log_{10}(x) \)

So, \( f'(x) = \cdots = \frac{12}{\ln(10) \cdot x} \)

(d) \( f(x) = 12 \ln(13) \)

Constant function

\( f'(x) = 0 \)

\( f'(x) = \frac{12}{13} ?? \)