- Pick up graded quizzes
- Sign in on the clipboard
- Exam 1 on Friday
- Study Guide is on course web page

Today: Continuing Section 2.7 Marginal Analysis

Estimation Problems

Derivatives of functions ("Marginal Quantities") can be used to estimate changes.
I'll explain by doing an example

2.7 # 34 A company makes guitars.

The variable \( x \), the "demand", is the number of guitars made in a batch.

The total cost of producing that batch of \( x \) guitars is

\[
C(x) = 1000 + 100x - 0.25x^2 \text{ dollars}
\]

(A) What is the total cost of producing a batch of 50 guitars?

Solution

\[
C(50) = 1000 + 100(50) - 0.25(50)^2
\]

\[
= 1000 + 5000 - 0.25(2500)
\]

\[
= 6000 - 625
\]

\[
= 5375
\]
(B) What is the total cost of producing a batch of 51 guitars?

Solution

\[ C(51) = 1000 + 100(51) - 0.25(51)^2 \]
\[ = 1000 + 5100 - 0.25(2601) \]
\[ = 6100 - 650.25 \]
\[ = 5449.75 \]

(C) If batch size changes from \( x = 50 \) to \( x = 51 \) guitars, what is the exact change in the cost of producing another?

Solution

\[ \Delta C = C(51) - C(50) = 5449.75 - 5375 = 74.75 \]

The book would call this “the cost of the 51\textsuperscript{st} guitar.” The book question says “Find the exact cost of producing the 51\textsuperscript{st} guitar.”
Illustration of question C1

$74.25 \quad \Delta C

\Delta x = 1

x = 5 \Delta \quad x = 5 \Delta
(D) Use the marginal cost function to find an approximate value for the change in cost of producing a batch of guitars when the batch size changes from $x = 50$ to $x = 51$ guitars.

Solution

\[ C(x) = 1000 + 100x - 0.25x^2 \]

\[ C(50) = 5375 \]

Marginal cost

\[ C'(x) = \frac{d}{dx}(1000 + 100x - 0.25x^2) = 100 - 0.5x \]

Marginal cost at a production level of 50:

\[ C'(50) = 100 - 0.5(50) = 100 - 25 = 75 \]

= slope of the line tangent to graph of $C(x)$ at the point where $x = 50$
The diagram shows a linear graph with the following annotations:

- The point (50, 5375) is marked.
- The point (51, 5449.25) is marked.
- The slope of the line is labeled as $m = 75$.
- The text "OC exact change hard to calculate" is marked in the diagram.
two triangles

hypotenuse is secant line

$\Delta x = 1$

$\Delta C = 74.75$

(50, 5375)

slope $m = 75$

hypotenuse is tangent line

$h$

$\Delta x = 1$

50, 5375

Let's try to find the height $h$.

Slope: $m = \frac{\Delta y}{\Delta x}$

$m = \frac{h}{\Delta x}$

Solve for $h$ by multiplying both sides by $\Delta x$

$h = m \cdot \Delta x = 75(1)$

$h = 75$
So the exact change is \( \Delta C = C(51) - C(50) = 74.25 \).

The approximate change is \( h = \max_x = C'(50) \cdot (1) = C'(50) = 75 \).
Be careful of equal signs

exact change = \Delta c = \$74.25

approximate change = h = \$75

exact change \cong approximate change
\Delta c \cong \$75
\Delta c \cong h
\Delta c \cong c'(50)
The reason this is attractive in general is
- the calculation of the marginal quantity is often much easier
  than the exact change calculation.
- The cost function is always just an approximate function anyway.
Price - Demand Equation

Example 2.7#4 Suppose price and demand for a particular item are related by the equation

\[ X = 1000 - 20p \]

Price demand equation

Find price \( p \) in terms of \( X \).

Solution Solve for \( p \).

\[ X + 20p = 1000 \]

\[ 20p = 1000 - X \]

\[ p = \frac{1000 - X}{20} = \frac{1000}{20} - \frac{X}{20} \]

\[ p = 50 - \left( \frac{1}{20} \right)X \]
b) What is the domain?

clearly must have \( x \geq 0 \). Can’t make a negative number of items.
but also must have \( p \geq 0 \) price can’t be negative.

End of Lecture