Fri Sep 4, 2015

- Quiz Ends at 12:05
- Write your name on the quiz
- National Calculus Study Day on Monday, No Class

Section 2.4 The Derivative

Today: Rates of Change
We'll do a series of examples involving

\[ f(x) = -x^2 + 8x - 7 = -(x-1)(x-7) \]
(A) Draw the graph on paper

\[ f(x) \]

\( (0, -1) \)
\( (1, 0) \)
\( (7, 0) \)
(B) Draw the secant line that passes through the points \((2, f(2))\) and \((4, f(4))\).

\[
f(2) = -(2-1)(2-7) = -(1)(-5) = 5
\]

\[
f(4) = -(4-1)(4-7) = -(3)(-3) = 9
\]
(c) Find the slope of that secant line.

\[ m = \frac{\Delta y}{\Delta x} = \frac{f(4) - f(2)}{4 - 2} = \frac{9 - 5}{4 - 2} = \frac{4}{2} = 2 \]

\[
\text{Difference Quotient}
\]

Average Rate of Change

See Reference 3 on page 3 of Course Packet

Example Continued

(d) Compute the average rate of change of \( f \)

from \( x = 2 \) to \( x = 2 + h \) when \( h \neq 0 \)

Solution: We have to build this

\[ m = \frac{f(2 + h) - f(2)}{(2 + h) - 2} = \frac{f(2 + h) - f(2)}{h} \]

\[
\text{Difference Quotient}
\]
\[ M = \frac{f(2+h) - f(2)}{h} \]

\[ = \frac{\left(- (2+h)^2 + 8(2+h) - 7\right) - \left(- (2)^2 + 8(2) - 7\right)}{h} \]

\[ = \frac{\left(- (4+4h+h^2) + 8(2) + 8h - 7\right) - \left(- 4 + 8(2) - 7\right)}{h} \]

\[ = \frac{-4 - 4h - h^2 + 8h - 7) - (- 4 + 8(2) - 7)}{h} \]

\[ = \frac{-4h - h^2 + 8h}{h} \]

\[ = 4h - h^2 \]
\[ f(x) = -x^2 + 8x - 7 \]
\[ = -(x)^2 + 8(x) - 7 \]
\[ f(\ ) = -(\ )^2 + 8(\ ) - 7 \]
\[ f(2+h) = -(2+h)^2 + 8(2+h) - 7 \]
\[ m = \frac{h(4-h)}{h} \]

We know \( h \neq 0 \) so we can cancel \( \frac{h}{h} \).

\[ m = 4-h \]

Slope of the secant line.

\[ (x, f(2+h)) \]

\[ (2, 5) \]

\[ x = 2 \quad x = 2+h \]
Example: If $h = 2$ then the two x-coordinates are $x_1 = 2$ and $x_2 = 2 + h = 2 + 2 = 4$.

That's the earlier example. The slope would be

$$m = \frac{4}{4-2} = 2$$

(This agrees with what we got before.)

Earlier example

\[ m = \frac{4-h}{4-2} = 2 \]