Wed Sept 2, 2015

Quiz will start as soon after 11:50 as possible, whenever all your stuff is put away.

When you finish, please turn your quiz face down and sit quietly. (No Phones.)

Quiz ends at 12:20
Studying sign behavior of functions

Function \( f \) can only change sign at these kinds of important \( x \)-values

- \( x \) value where \( f(x) = 0 \) (\( x \)-intercept)
- \( x \) value where \( f \) is discontinuous

Define these to be the Partition Numbers for \( f \)

In between partition numbers, the sign of \( f \) does not change.
Example for $f(x) = 9x^2 - 90x + 189$

Determine the sign behavior of $f$.

Solution: find the partition numbers.

Notice $f$ is polynomial, so it is always continuous.
The only partition numbers will be the $x$-coordinate of the $x$-intercepts.
That is find all $x$ such that $f(x) = 0$

Turn this around

$0 = f(x)$

$= 9x^2 - 90x + 189$

$= 9(x^2 - 10x + 21)$  

$= 9(x - 3)(x - 7)$  

factor out a 9.
Check: 
\[(x-3)(x-7) = x^2 - 3x - 7x + 21\]
\[= x^2 - 10x + 21\]

\[9(x-3)(x-7) = 9(x^2 - 10x + 21)\]
\[= 9x^2 - 90x + 189\]  

So partition numbers are \[x = 3\] \[x = 7\]

\[x = 9\]  
Not a partition number. It does not cause \(f(x)\) to be 0.
Sign chart for $f(x) = 9x^2 - 90x + 189 = 9(x-3)(x-7)$

<table>
<thead>
<tr>
<th>Important intervals and x-values</th>
<th>$9$</th>
<th>Sign of factors</th>
<th>Sign of $f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x &lt; 3$</td>
<td>$+$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
<tr>
<td>$3 &lt; x &lt; 7$</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>$7 &lt; x$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td>Partition $x = 3$</td>
<td>$+$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Partition $x = 7$</td>
<td>$+$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>
(B) Solve the inequality \( 9x^2 - 90x + 189 \geq 0 \)

Solution: Using sign chart

\[ x \leq 3 \text{ or } 7 \leq x \]

\[ (-\infty, 3] \cup [7, \infty) \]

Example 2: Solve the inequality

\[ 9x^2 - 90x \geq -189 \]
Solution: Add 189 to both sides

\[ 9x^2 - 90x + 189 = 0 \]

This is the problem that we just solved.

Example: Solve the inequality \( \frac{x^2 - 5x}{x - y} \leq 0 \)

Solution: Factor the function

\[ \frac{x^2 - 5x}{x - y} = \frac{x(x-5)}{x - y} \]

Partition numbers:

- \( x = 0 \) causes \( y = 0 \)
- \( x = 5 \) causes \( y = 0 \)
- \( x = y \) is discontinuous there
Sign chart for $f(x) = \frac{x(x-5)}{x-4}$

<table>
<thead>
<tr>
<th>$x$-values</th>
<th>$x$</th>
<th>$x-5$</th>
<th>$f(x)$</th>
<th>$f(x)$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x &lt; 0$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>neg</td>
</tr>
<tr>
<td>$x = 0$</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>$0 &lt; x &lt; 4$</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>pos</td>
</tr>
<tr>
<td>$x = 4$</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td>DNE</td>
</tr>
<tr>
<td>$4 &lt; x &lt; 5$</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>neg</td>
</tr>
<tr>
<td>$x = 5$</td>
<td>+</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>$5 &lt; x$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>pos</td>
</tr>
</tbody>
</table>
Solution to the inequality $f(x) \leq 0$ is $x = 0$ or $4 < x \leq 5$.

Inequality notation: $x \leq 0$ or $4 < x \leq 5$

Interval notation: $(-\infty, 0] \cup (4, 5]$