Tues Sept 1, 2015

Section 2.3 Continuity

Informal definition of Continuity

at $x = a$ \( \{ \text{A function } f \text{ is continuous at } x = a \text{ if the graph of } f \text{ near } x = a \text{ can be drawn without lifting your pen.} \) 

on an interval \( \{ \text{A function } f \text{ is continuous on an interval } [a, b] \text{ if the graph can be drawn from } x = a \text{ to } x = b \text{ without lifting your pen.} \) 

Official Definition of Continuity at $x = a$

words: $f$ is continuous at $x = a$

meaning: $f$ passes this three-part test

**Test 1** $\lim_{x \to a} f(x)$ must exist

**Test 1a** $\lim_{x \to a^-} f(x)$ must exist

**Test 1b** $\lim_{x \to a^+} f(x)$ must exist

**Test 1c** The values of the limits in 1a, 1b must match

**Test 2** $f(a)$ must exist

**Test 3** The values from test 1 to test 2 must match

Do Class Drill 4
Class Drill 4: Limits and Continuity of a Function Given by a Graph

Use the graph below to answer the questions that follow.

(1) For each asymptote, give the line equation and say whether it is horizontal or vertical.

\[ y = 3 \text{ horizontal} \]
\[ y = 1 \text{ horizontal} \]
\[ x = 2 \text{ vertical} \]

(2) \[ \lim_{x \to -\infty} f(x) = 3 \text{ because graph has horizontal asymptote at } y = 3 \]

(3) \[ \lim_{x \to -5} f(x) = \text{DNE (left & right limits don't match)} \]
\[ \lim_{x \to -5^-} f(x) = 3 \]
\[ \lim_{x \to -5^+} f(x) = 2 \]

(4) \[ \lim_{x \to 2^-} f(x) = -1 \text{ because graph heading for } (x, y) = (-2, -1) \]

(5) \[ \lim_{x \to 2} f(x) = \infty \]

(6) \[ \lim_{x \to 6} f(x) = 3 \text{ because graph heading for } (x, y) = (6, 3) \]

(7) \[ \lim_{x \to \infty} f(x) = 1 \text{ because graph has horizontal asymptote on right at } y = 1 \]
(8) Is \( f \) continuous at \( a = -5 \)? If not, explain why not.

**No. (flunks test 1(c))**

(9) Is \( f \) continuous at \( a = -2 \)? If not, explain why not.

**Yes, passes all 3 tests**

(10) Is \( f \) continuous at \( a = 2 \)? If not, explain why not.

**No. Flunks test 2**

(11) Is \( f \) continuous at \( a = 6 \)? If not, explain why not.

**No. Flunks test 3.**

Remember that for a function \( f \) to be continuous at some number "\( a \)”, the function must pass these three tests:

Test 1: \( \lim_{x \to a} f(x) \) must exist

Test 1a: \( \lim_{x \to a^-} f(x) \) must exist

Test 1b: \( \lim_{x \to a^+} f(x) \) must exist

Test 1c: The numbers in test 1a and 1b must agree.

Test 2: \( f(a) \) must exist

Test 3: The numbers in test 1 and test 2 must agree.
Official definition of continuous on an interval

Words: \( f \) is continuous on interval \((a, b)\)

Meaning: \( f \) is continuous at every \( x = c \) where \( a < c < b \)

\( f \) passes the 3-part test at every \( x \)-value in the interval

\( f \) does not fail the continuity test anywhere on the interval.

Observation: Polynomial functions are continuous everywhere.

\( y = x^2 \)

(This is a theorem.)
That tells us that for polynomial functions
\[ \lim_{x \to c} f(x) = f(c) \] for all \( c \).

This is Limit Law Theorem 3 from Section 2.1. That theorem comes from the theory of continuity.

Rational functions \( f(x) = \frac{\text{polynomial}}{\text{polynomial}} \)
are continuous everywhere except at \( x \)-values that cause denominator \( = 0 \).

So for rational functions, the equation \( \lim_{x \to c} f(x) = f(c) \) is always true except for \( x \)-values that cause denominator \( = 0 \). That is, the other part of Limit Law Theorem 3.
Sign behavior of functions

Notice

Functions can change sign at x-values that are
- x-intercepts
- x-values where f is discontinuous.

In between those x-values, the sign of the function does not change.

end of lecture