Continuing Section 2.2 Limits involving Infinity.

Consider function from Class Drill 3

\[ f(x) = \frac{x^2 - 6x + 5}{x^2 - 8x + 15} = \frac{(x - 1)(x - 5)}{(x - 3)(x - 5)} \]

Standard form \text{ Factored form}

Consider what the factored form tells us,

\( x = 1, 3, 5 \) are “important” (and easy to spot) because they cause the factors to be zero.

Consider the corresponding y-values

\[ f(1) = \frac{(1 - 1)(1 - 5)}{(1 - 3)(1 - 5)} = \frac{0 \cdot (-4)}{-2 \cdot (-4)} = \frac{0}{8} = 0 \]

So \((x, y) = (1, 0)\) is a point on the graph. An x-intercept.
\[ f(3) = \frac{(3-1)(3-5)}{(3-3)(3-5)} = \frac{2(-2)}{0(-2)} = \frac{-4}{0} \quad \text{undefined} \]

No y-value on graph at \( x=3 \)

\[ f(5) = \frac{(5-1)(5-5)}{(5-3)(5-5)} = \frac{4(0)}{2(0)} = \frac{0}{0} \quad \text{undefined} \]

No y-value on graph at \( x=5 \) either.

Study what goes on near \( x=3 \) and near \( x=5 \).

Class Drill 3 explored this, by plugging in numbers

we find \( \lim_{x \to 5} f(x) = 2 \)

So graph is heading for the location \( (x,y) = (5,2) \)

That is, since \( f(5) \) DNE we know

that graph has hole at \( (x,y) = (5,2) \).

So the factors \( \frac{x-5}{x-5} \) cause a hole in graph at \( x=5 \).
Now consider behavior near $x = 3$

Find $\lim_{x \to 3^-} f(x)$ and $\lim_{x \to 3^+} f(x)$

we found $\lim_{x \to 3^-} f(x) = -\infty$ and $\lim_{x \to 3^+} f(x) = \infty$

But $\lim_{x \to 3} f(x)$ DNE because left + right limits don't match.

Conclusion the factor $\frac{1}{x-3}$ causes a vertical asymptote at $x = 3$.

Graph goes down along left side of asymptote

Graph goes up along right side of asymptote.
Remark: The \( \lim_{x \to 5} f(x) = 2 \) is a limit that we could have done using section 2.1 techniques.

\[
\lim_{x \to 5} f(x) = \lim_{x \to 5} \frac{(x-5)(x-1)}{(x-5)(x-3)}
\]

\[
= \lim_{x \to 5} \frac{x-1}{x-3}
\]

\[
= \frac{5-1}{5-3}
\]

\[
= \frac{4}{2}
\]

\[
= 2 \quad \text{Same result as Wednesday}
\]
Now consider "Limits at Infinity"

Let \( f(x) = \frac{7x^2 - 42x + 35}{2x^2 - 16x + 30} \) 

Find \( \lim_{x \to \infty} f(x) \)

\[
\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{7x^2 - 42x + 35}{2x^2 - 16x + 30}
\]

\( x \to \infty \) means \( x \) gets more and more positive \underline{without bound}.

\[
= \lim_{x \to \infty} \frac{7x^2}{2x^2}
\]

\[
= \lim_{x \to \infty} \frac{7}{2}
\]

\[
= \frac{7}{2}
\]

\( x \) is huge

\[
\frac{7}{2}
\]

This tells us that as \( x \) gets more and more positive \underline{without bound}, the \( y \)-value gets closer and closer to \( y = \frac{7}{2} \).