Wed Aug 26, 2015

Quiz: Please put away all books + notes. Turn cell phone ringers off + stow away.

Review yesterday's example

Function \( f(x) = \frac{x^2 - 2x - 3}{x - 3} = \frac{(x-3)(x+1)}{(x-3)} \)

\( f(x) = \begin{cases} 
  x + 1 & \text{when } x \neq 3 \\
  \text{undefined} & \text{when } x = 3
\end{cases} \)

Compare that to the function \( g(x) = x + 1 \)

Domain: all \( x \neq 3 \)

Domain: all real numbers

Graphs:

- For \( f(x) \):
  - Hole at (3, 4)
  - Line: \( y = 4 \)
  - Points: (0, 1), (1, 2)

- For \( g(x) \):
  - Line: \( y = x + 1 \)
  - Points: (0, 1), (1, 2)
  - \( g(3) = 4 \)
So yesterday's example $f(3) = \frac{0}{0}$ DNE

$$\lim_{{x \to 3}} f(x) = 4$$

Section 2.2 Limits Involving Infinity

Consider the function $f(x) = \frac{1}{(x-3)^2}$

Find the y-value $f(3)$

Solution $f(3) = \frac{1}{(3-3)^2} = \frac{1}{0^2} = \frac{1}{0}$ DNE

Find the limit $\lim_{{x \to 3}} f(x)$ using Section 2.1 techniques

$$\lim_{{x \to 3}} f(x) = \lim_{{x \to 3}} \frac{1}{(x-3)^2}$$

Observe limit of numerator is $\lim_{{x \to 3}} 1 = 1$

Limit of denominator is $\lim_{{x \to 3}} (x-3)^2 = 0$

So limit does not exist using Section 2.1 techniques. (by Theorem 7)
But consider y-values for x values near 3.

<table>
<thead>
<tr>
<th>x</th>
<th>( f(x) = \frac{1}{(x-3)^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>( f(3.1) = \frac{1}{(3.1-3)^2} = \frac{1}{(0.1)^2} = \frac{1}{0.01} = 100 )</td>
</tr>
<tr>
<td>3.01</td>
<td>( f(3.01) = \frac{1}{(3.01-3)^2} = \frac{1}{(0.01)^2} = \frac{1}{0.0001} = 10,000 )</td>
</tr>
<tr>
<td>3.001</td>
<td>( f(3.001) = \frac{1}{(3.001-3)^2} = \frac{1}{(0.001)^2} = \frac{1}{0.000001} = 1,000,000 )</td>
</tr>
</tbody>
</table>

Notice as \( x \to 3^+ \), the values of \( f(x) \) get more and more positive, without bound.

**Abbreviation:** \( \lim_{x \to 3^+} f(x) = \infty \)

**Spoken:** "the limit, as \( x \) approaches 3 from the right, of \( f(x) \) is infinity"
Similarly find \( \lim_{{x \to 3^-}} f(x) \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.9</td>
<td>( f(2.9) = \frac{1}{(2.9-3)^2} = \frac{1}{(-.1)^2} = \frac{1}{.01} = 100 )</td>
</tr>
<tr>
<td>2.99</td>
<td>( f(2.99) = \cdots = 10,000 )</td>
</tr>
<tr>
<td>2.999</td>
<td>( f(2.999) = \cdots = 1,000,000 )</td>
</tr>
</tbody>
</table>

Conclude \( \lim_{{x \to 3^-}} f(x) = \infty \) as well. \( f(x) = \frac{1}{(x-3)^2} \)

Conclude \( \lim_{{x \to 3}} f(x) = \infty \).

Remember: under old definition of limit, the limit did not exist.

Now Do Class Drill 3
Class Drill 3: Guessing Limits by Substituting in Numbers

Without using a calculator, answer the following questions about the function

\[ f(x) = \frac{x^2 - 6x + 5}{x^2 - 8x + 15} \]

Part 1: Function Values

(1) Factor \( f \). (Check your factorizations by multiplying.)

\[ f(x) = \frac{(x-5)(x-1)}{(x-5)(x-3)} \]

Check: \( (x-5)(x-1) = x^2 - 5x - x + 5 = x^2 - 6x + 5 \)

(\( x-5)(x-3) = x^2 - 5x - 3x + 15 = x^2 - 8x + 15 \)

(2) Are you allowed to cancel factors in the factored form of \( f \)? Explain why you think you are allowed to cancel, or why you are not.

Cannot cancel factors, because we don't know the value of \( x \), so we don't know if \( x-5 \neq 0 \).

(3) Find \( f(1) \) by substituting \( x = 1 \) into the factored version of \( f \).

\[ f(1) = \frac{(1-5)(1-1)}{(1-5)(1-3)} = \frac{0}{-2} = 0 \]

(4) Find \( f(3) \) by substituting \( x = 3 \) into the factored version of \( f \).

\[ f(3) = \frac{(3-5)(3-1)}{(3-5)(3-3)} = \frac{2}{0} \text{ DNE} \]

(5) Find \( f(5) \) by substituting \( x = 5 \) into the factored version of \( f \).

\[ f(5) = \frac{(5-5)(5-1)}{(5-5)(5-3)} = \frac{0(4)}{0(2)} = 0 \text{ DNE} \]

Part 2: Limits

Using the factored form of \( f \), compute the following values and guess the limits.

Guessing the limit at \( x = 5 \).

(Just leave answers as an expression ready to type into a calculator.)

(11) \( f(5.1) = \frac{(5.1-5)(5.1-1)}{(5.1-5)(5.1-3)} = \frac{4.1}{2.1} \)

(12) \( f(5.01) = \frac{(5.01-5)(5.01-1)}{(5.01-5)(5.01-3)} = \frac{4.01}{2.1} \)

(13) \( f(5.001) = \frac{(5.001-5)(5.001-1)}{(5.001-5)(5.001-3)} = \frac{4.001}{2.001} \)

(15) \( \lim_{x \to 5^+} f(x) = 2 \because \text{numerator} \to 4 \text{ while denominator} \to 2 \)
(16) \( f(4.9) = \frac{(4.9 - 5)(4.9 - 1)}{(4.9 - 5)(4.9 - 3)} = \frac{3.9}{1.9} \)

(17) \( f(4.99) = \frac{(4.99 - 5)(4.99 - 1)}{(4.99 - 5)(4.99 - 3)} = \frac{3.99}{1.99} \)

(18) \( f(4.999) = \frac{(4.999 - 5)(4.999 - 1)}{(4.999 - 5)(4.999 - 3)} = \frac{3.999}{1.999} \)

(20) Guess \( \lim_{x \to 5^-} f(x) = 2 \) because numerator \( \to 4 \) and denominator \( \to 2 \)

(21) Guess \( \lim_{x \to 5} f(x) = 2 \) because the left and right limits are both 2.

**Guessing the limit at \( x = 3 \). (Simplify your answers.)**

(11) \( f(3.1) = \frac{(3.1 - 5)(3.1 - 1)}{(3.1 - 5)(3.1 - 3)} = \frac{2.1}{0.1} = 21 \)

(12) \( f(3.01) = \frac{(3.01 - 5)(3.01 - 1)}{(3.01 - 5)(3.01 - 3)} = \frac{2.01}{0.01} = 201 \)

(13) \( f(3.001) = \frac{(3.001 - 5)(3.001 - 1)}{(3.001 - 5)(3.001 - 3)} = \frac{2.001}{0.001} = 2001 \)

(15) Guess \( \lim_{x \to 3^+} f(x) = \infty \) because the values are getting more positive without bound.

(16) \( f(2.9) = \frac{(2.9 - 5)(2.9 - 1)}{(2.9 - 5)(2.9 - 3)} = \frac{-1.9}{-1} = -19 \)

(17) \( f(2.99) = \frac{(2.99 - 5)(2.99 - 1)}{(2.99 - 5)(2.99 - 3)} = \frac{1.99}{-0.01} = -199 \)

(18) \( f(2.999) = \frac{(2.999 - 5)(2.999 - 1)}{(2.999 - 5)(2.999 - 3)} = \frac{1.999}{-0.001} = -1999 \)

(20) Guess \( \lim_{x \to 3^-} f(x) = -\infty \) because the y-values are getting more negative without bound.

(21) Guess \( \lim_{x \to 3} f(x) = \text{DNE} \) because the left and right limits don't match.