Cover Sheet for 2014-2015 Spring Semester MATH 3210/5210 (Barsamian) Homework 7
(Due at the start of class Wednesday, March 18, 2014. Staple this cover sheet to the front of your work.)

<table>
<thead>
<tr>
<th>Problem:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Total</th>
<th>Rescaled</th>
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</thead>
<tbody>
<tr>
<td>Your Score:</td>
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<td>Possible:</td>
<td>10</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>10</td>
<td>20</td>
<td>100</td>
<td>10</td>
</tr>
</tbody>
</table>

Note that whenever the exercise numbering or page numbering is different for the new and old editions of the book, the information for the old edition of the book is shown in (parentheses).

**Reading:** In Chapter Three, Maps Between Spaces, read the following subsections:

Three.II.2 Homomorphisms: Range Space and Null Space
The helpful pages are 189 (191) through example 2.4 on page 190 (192), then resume reading with Lemma 2.10 on page 194 (196), and read through page 198 (200)

**Suggested Exercises:** Three.II.2 # 21, 22, 25(23), 26(24), 28(26), 33(31), 35(33), 38(36) p 198-201 (200-202)

[1] Define map \( f: \mathcal{P}_2 \to \mathcal{P}_3 \) by \( f(p(x)) = x \cdot p(x) \). For example, \( f(5 + 4x - x^2) = x \cdot (5 + 4x - x^2) \)
Which of these vectors are in the range space of \( f \)? Explain. (If you think a vector is in the range space, you have to give an example of an input vector that will produce that vector as an output.)

(a) \( \bar{v}_a = x^2 \)  
(b) \( \bar{v}_b = 2x + 13x^2 \)  
(c) \( \bar{v}_c = 2 + 13x^2 \)  
(d) \( \bar{v}_d = 0 \)  
(e) \( \bar{v}_e = 2 \)

For each map in problems [2], [3], [4], answer the following:
(a) Is the map onto? (Explain)
(b) Find the range space. (Explain) (The definition of range space is on page 192.)
(c) Find a basis for the range space. (Explain)
(d) Find the rank of the map. (Explain) (The definition of range space is on page 192.)

[2] The map \( f: \mathcal{P}_2 \to \mathcal{P}_3 \) by \( f(p(x)) = x \cdot p(x) \) that was introduced in problem [1].

[3] The differentiation map \( D: \mathcal{P}_2 \to \mathcal{P}_2 \) defined by \( D(f) = \frac{d}{dx} f(x) \)

[4] The map \( f: \mathbb{R}^2 \to \mathcal{P}_2 \) defined by \( f \left( \begin{array}{c} a \\ b \end{array} \right) = 2bx - 5bx^2 \).

[5] The map \( g: \mathcal{M}_{2\times 2} \to \mathbb{R} \) defined by \( f \left( \begin{array}{cc} a & b \\ c & d \end{array} \right) = a + b + c + d \).