Cover Sheet for 2014-2015 Fall Semester MATH 3200/5200 (Barsamian) Homework 8
(Do at the start of class Friday, November 7, 2014. Staple this cover sheet to the front of your work.)

<table>
<thead>
<tr>
<th>Problem:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Total</th>
<th>Rescaled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Possible:</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>100</td>
<td>10</td>
</tr>
</tbody>
</table>

Reading: In Chapter Three, Maps Between Spaces, read the following subsections:
Three.III.1: Computing Linear Maps: Representing Linear Maps with Matrices (p. 203-210)
Three.III.2: Computing Linear Maps: Any Matrix Represents a Linear Map (p. 213-218)

Suggested Exercises: Three.III.1 Exercises #12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 28 (p. 210-213)
Three.III.2 Exercises #12, 16, 20, 21, 22, 23, 26 (p. 218-221)

Let \( \alpha, \beta, \gamma \) be the standard bases for \( \mathbb{R}, \mathcal{P}_2, \mathcal{P}_3 \). That is, let \( \alpha = (1) \) and \( \beta = (1, x, x^2) \) and \( \gamma = (1, x, x^2, x^3) \). And let \( \delta \) be the non-standard basis \( \delta = (1, 2x, 3x^2) \) for \( \mathcal{P}_2 \).

1] Let \( D \) be the “derivative, restricted to the 3rd degree polynomials”. That is, \( D \) is the map \( D: \mathcal{P}_3 \to \mathcal{P}_2 \) defined by \( D(f) = f' \).
   (a) Find \( \text{Rep}_{\gamma,\beta}(D) \).
   (b) Find \( \text{Rep}_{\gamma,\delta}(D) \).

2] Let \( I_{[0,x]} \) be the map “definite integral from 0 to \( x \), restricted to the 2nd degree polynomials”. That is, \( I_{[0,x]} \) is the map \( I_{[0,x]}: \mathcal{P}_2 \to \mathcal{P}_3 \) defined by
   \[
   I_{[0,x]}(f) = \int_{t=0}^{t=x} f(t)dt
   \]
   (a) Find \( \text{Rep}_{\beta,\gamma}(I_{[0,x]}) \).
   (b) Find \( \text{Rep}_{\delta,\gamma}(I_{[0,x]}) \).

3] Let \( \text{eval}_5 \) denote the “evaluation at 5 map, restricted to the 3rd degree polynomials”. That is, \( \text{eval}_5 \) is the map \( \text{eval}_5: \mathcal{P}_3 \to \mathbb{R} \) defined by \( \text{eval}_5(f) = f(5) \). Represent the map \( \text{eval}_5 \) with respect to the bases \( \gamma \) and \( \alpha \).

4] Let \( h: \mathcal{P}_2 \to \mathcal{P}_3 \) be a linear map such that \( h(1) = x + x^2 \) and \( h(x) = 1 - 2x \) and \( h(x^2) = 3 + x - x^2 \), and let \( \vec{v} = 2 - x + 3x^2 \in \mathcal{P}_2 \). The goal is to find \( h(\vec{v}) \) using matrix operations.
   (a) Find the matrix \( \text{Rep}_{\beta,\gamma}(h) \).
   (b) Find the column vector \( \text{Rep}_{\beta}(\vec{v}) \).
   (c) Using the matrix-vector product, find \( \text{Rep}_{\gamma}(h(\vec{v})) = \text{Rep}_{\beta,\gamma}(h) \cdot \text{Rep}_{\beta}(\vec{v}) \).
   (d) Using your result from (c), find \( h(\vec{v}) \).

5] Suppose that \( \text{Rep}_{\beta,\delta}(f) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \). Observe that this is the 3 \( \times \) 3 identity matrix.
   (a) What is the domain of map \( f \)? What is the codomain of \( f \)?
   (b) Find \( f(1 - 3x + 2x^2) \). The result should be a function.
   (Observe that \( f \) is not the identity map, because \( f(1 - 3x + 2x^2) \neq 1 - 3x + 2x^2 \))

Now let \( \text{id}: \mathcal{P}_2 \to \mathcal{P}_2 \) be the identity map. That is, \( \text{id}(ax + bx + cx^2) = ax + bx + cx^2 \).
   (c) Find \( \text{Rep}_{\beta,\delta}(\text{id}) \).
   (d) Find \( \text{Rep}_{\beta,\delta}(\text{id}) \). (Observe that the result is not the 3 \( \times \) 3 identity matrix!)

Note: Part (d) originally said \( \text{Rep}_{\beta,\gamma}(\text{id}) \). That was a typo!