Cover Sheet for 2014-2015 Fall Semester MATH 3200/5200 (Barsamian) Homework 1
(Due at the start of class on Friday, August 29, 2014. Staple this cover sheet to the front of your work.)

<table>
<thead>
<tr>
<th>Problem</th>
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<th>5</th>
<th>Total</th>
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<tr>
<td>Possible:</td>
<td>20</td>
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<td>100</td>
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Reading: In Chapter One, read Sections One.I.1 and One.I.2, pages 1 – 22.

Suggested Exercises: (These seventeen exercises are not to be turned in and are not graded, but you should do as many of them as possible and keep your solutions in a notebook for study. Note that detailed solutions to all of the Suggested Exercises are available in the solutions manual provided for free on the author’s web site.)

Section One.I.1 Exercises # 18, 20, 23, 29, 30, 32, 33, 35 (from pages 9 – 12)
Section One.I.2 Exercises # 18, 20, 21, 22, 23, 24, 26, 27, 30 (from pages 19 – 22)

Assigned Exercises: Turn in your solutions to the following five exercises, with this cover sheet stapled to the front of your work.

[1] (20 points) (Similar to One.I.1#18) Use Gauss’s method to solve each system or conclude ‘no solution’ or ‘many solutions’. Show all steps clearly.

(a) \( \begin{cases} 2x + 3y = 3 \\ x - y = 4 \end{cases} \)  
(b) \( \begin{cases} 2x + 3y = 3 \\ 4x + 6y = 5 \end{cases} \)  
(c) \( \begin{cases} x - y - z = 1 \\ 4x - 2y - z = 5 \end{cases} \)  
(d) \( \begin{cases} 2x + z = 3 \\ x - y - z = 1 \\ 3x - y = 4 \end{cases} \)

[2] (20 points) (Similar to One.I.1#23) True or False: A system with four unknowns and three equations always has many solutions. You must justify your answer with a proof or a counterexample!

[3] (20 points) (Similar to One.I.2#18) Solve each system using matrix notation. Express the solution set using vectors. Show all steps clearly.

(a) \( \begin{cases} 2x - y + 3z = 3 \\ x - 2y - z = 3 \end{cases} \)  
(b) \( \begin{cases} x + z = 1 \\ 2x - y + 3z = 3 \\ 3x - y + 4z = 4 \end{cases} \)  
(c) \( \begin{cases} x + z = 1 \\ x + z = 1 \\ 2x - y + 3z = 3 \\ 3x - y + 4z = 5 \end{cases} \)

[4] (20 points) (Similar to One.I.2#21) Decide if the vector is in the set. Justify your answers.

(a) Is the vector \( \begin{pmatrix} 2 \\ -3 \end{pmatrix} \) in the set \( \{ (\begin{pmatrix} 4 \\ -6 \end{pmatrix} k | k \in \mathbb{R} ) \} \)?

(b) Is the vector \( \begin{pmatrix} 2 \\ 5 \end{pmatrix} \) in the set \( \{ (\begin{pmatrix} -6 \\ 15 \end{pmatrix} j | j \in \mathbb{R} ) \} \)?

(c) Is the vector \( \begin{pmatrix} -5 \\ -3 \\ 1 \end{pmatrix} \) in the set \( \{ (\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} + (\begin{pmatrix} 2 \\ 1 \end{pmatrix} ) r | r \in \mathbb{R} ) \} \)?

(d) Is the vector \( \begin{pmatrix} -2 \\ 4 \end{pmatrix} \) in the set \( \{ (\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} j + (\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} ) k | j, k \in \mathbb{R} ) \} \)?

[5] (20 points) (Similar to One.I.2#30) Make up a three equations / three unknowns system having

(a) no solutions.  
(b) exactly one solution.  
(c) a one-parameter solution set.  
(d) a two-parameter solution set.

Either explain your answers, or provide answers that are so clear that they do not need explanations.