Cover Sheet for 2013-2014 Spring Semester MATH 3210/5210 (Barsamian) Homework 7
(Due Friday, April 4, 2014. Staple this page to the front of your work.)

<table>
<thead>
<tr>
<th>Problem</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Total</th>
<th>Rescaled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Your Score:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Possible:</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>100</td>
<td>10</td>
</tr>
</tbody>
</table>

[1] (20 points) (Five True False questions from suggested exercise 4.4#1.) The True/False answers for some of the questions are given in the back of the book. For each question, give the true/false answer and also explain why the answer is true or false. Justify your explanation by referring to things that the book says in Section 4.4.

4.4#1 (c) If two rows or columns of a matrix $A$ are identical, then $\det(A) = 0$.
4.4#1 (g) The determinant of an upper-triangular $n \times n$ matrix is the product of its diagonal entries.
4.4#1 (i) If $A, B \in M_{n \times n}(F)$, then $\det(AB) = \det(A)\det(B)$.
4.4#1 (j) If $Q$ is an invertible matrix, then $\det(Q^{-1}) = (\det(A))^{-1}$.
4.4#1 (k) A matrix $Q$ is invertible if and only if $\det(Q) \neq 0$.

[2] (20 points) (Similar to suggested exercise 4.4#2(b),(d)) Evaluate the determinant of the following matrices. Show all details of the calculations.

(a) $A = \begin{pmatrix} 3 & -5 \\ 1 & 2 \end{pmatrix}$
(b) $b = \begin{pmatrix} 5i \\ -3 \\ 2i \end{pmatrix}$

[3] (20 points) (Similar to suggested exercise 4.4#3(d)) Evaluate the determinant along the third row. Show all details of the calculation.

$$A = \begin{pmatrix} 2 & 0 & 7 \\ 3 & -1 & 1 \\ 0 & 2 & -1 \end{pmatrix}$$

[4] (20 points) (Similar to suggested exercise 4.4#5) Suppose that a matrix $A \in M_{n \times n}(F)$ can be written in the form $A = \begin{pmatrix} P & Q \\ 0 & I \end{pmatrix}$ where $P, Q, 0, I$ are square matrices, 0 is the zero matrix, and $I$ is the identity matrix. Prove that $\det(A) = \det(P)$. Justify the statements of your proof using information from Section 4.4 of the book.

[5] (20 points) (Similar to suggested exercise 5.1#2(b)) Let $V$ be the vector space $V = P_1(R)$ with basis $\beta = \{v_1, v_2\} = \{3 + 2x, 4 + 3x\}$

Let $T: P_1(R) \rightarrow P_1(R)$ be the linear transform $T(a + bx) = (-11a + 12b) + (-6a + 6b)x$.

(a) Compute $[T]_{\beta}$. Show all details of the calculation and explain your steps.

(b) Are the basis vectors $v_1, v_2$ eigenvectors for $T$? Explain why.

(c) If the vectors $v_1, v_2$ are eigenvectors for $T$, then what are the eigenvalues? Explain.