Reference 7: Definitions of Rates of Change

Definition of *Average Rate of Change*
- **words:** the average rate of change of $f$ as the input changes from $a$ to $b$
- **usage:** $f$ is a function that is continuous on the interval $[a, b]$.
- **meaning:** the number $m = \frac{f(b)-f(a)}{b-a}$
- **graphical interpretation:** The number $m$ is the slope of the secant line that touches the graph of $f$ at the points $(a, f(a))$ and $(b, f(b))$.
- **remark:** The average rate of change $m$ is a number.

Definition of *Instantaneous Rate of Change*
- **words:** the instantaneous rate of change of $f$ at $a$
- **alternate words:** the derivative of $f$ at $a$
- **symbol:** $f'(a)$
- **meaning:** the number $m = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$
- **graphical interpretation:** The number $m$ is the slope of the line tangent to the graph of $f$ at the point $(x, y) = (a, f(a))$.
- **remark:** The instantaneous rate of change $f'(a)$ is a number.

Definition of the *Derivative*
- **words:** the derivative of $f$
- **symbol:** $f'$
- **meaning:** $f'$ is a function. To describe a function, one must show how it produces output for a given input. For an input $x$, the output is the number $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.
- **graphical interpretation:** For an input $x$, the output $f'(x)$ is the number that is the slope of the line tangent to the graph of $f$ at the point $(x, y) = (x, f(x))$.
- **remark:** The derivative $f'$ is a function.

Terminology of *Position and Velocity*
- **Time:** When our book uses mathematical functions to describe the motion of objects, $x$ is a variable that represents the elapsed time.
- **Position:** To say an object is “moving in 1 dimension” means that it can go forward or backward in one direction but cannot turn. In such situations, a single coordinate can be used to keep track of the position of the object. A function called the *position function* gives the value of the coordinate at a given time. In our book, the position function is called $f$. That is, at time $x$, the coordinate of the object is the number $f(x)$.
- **average velocity:** The words “the average velocity from time $x = a$ to time $x = b$ ” mean the same thing as “the average rate of change of position from time $x = a$ to time $x = b$ ”. That is, the number $m = \frac{f(b)-f(a)}{b-a}$.
- **instantaneous velocity:** The words “instantaneous velocity at time $x = a$ ” mean the same thing as “instantaneous rate of change of position at time $x = a$.” That is, the number $m = f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$.
- **velocity:** The word “velocity” means the same thing as the words “derivative of the position function”. That is, the velocity is the function $f'$. 